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Three Phenomenological Potentials for the a-a Interaction*

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Three phenomenological effective α-α potentials, soft-core square well, hard-core square well and Eckart are constructed by a least squares fit of the *a-a* cross-section for low energy The theoretical method consists of matching wavefunctions in a region where only the Coulomb interaction is dominant Only the *s*, *d* and *g* partial waves were required to reproduce the experimental *a-a* cross-section.

Constroem-se três potenciais fenomenológicos efetivos, fazendo-se um ajuste deminuminimo erro quadrático da seção de choque α-α a baixa energia, poço quadrado com caroço mole, poço quadrado com caroço duro e potencial de Eckart. O método teórico consiste em obter concordância de funções de onda numa região onde somente a interação coulombiana e dominante. Bastaram apenas as ondas parciais *s*, *d* e *g* para reproduzir a seção de choque α-α experimental.

1. Introduction

Various researchers have tried to construct a-a potentials from first principles using the Resonating Group Method[1]. According to this scheme, one begins with a treatment of all of the nucleons, assumed to interact via 2-body forces, which can be written as a sum of three terms: a direct potential, a nucleon exchange potential anda polarization potential. For systems of 4n nucleons, the total wavefunction is then transformed so that its spatial dependence clearly exhibits an α structure. For example, in ^{8}Be, one has

$$\Psi = A[\Phi_1 \Phi_2 \varphi_\alpha(\mathbf{r}) X(\sigma, \tau)], \qquad (1)$$

where A is an antisymmetrization operator, Φ_1 and Φ_2 the intrinsic wavefunctions describing the spatial behavior of the a-clusters, and χ an appropriate charge-spin function. The function φ_α describes the relative mo-

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tion of the clusters ($r = r, -\mathbf{r}_2$) and is determined from the variational principle

$$\delta \int \Psi^*(H-E)\,\Psi\, dV = 0, \tag{2}$$

where H is the Hamiltonian of the α-α system and E is the total energy, which is composed of the interna1 energies of the clusters and the relative energy in the C.M. system. The variational calculation leads to the following integro-differential equation

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_D(r) + V_C(r) - \varepsilon\right]\varphi_{l,\varepsilon}(r) =$$

$$= -\int_0^\infty K_{ll}(r', r)\,\varphi_{l,\varepsilon}(r')\,dr' \tag{3}$$

where

$$\varphi_\alpha(\mathbf{r}) = \sum_{lm}\frac{\varphi_{l,\varepsilon}(r)}{r}\,Y_{lm}(\hat{\mathbf{r}}), \tag{4}$$

and μ is the reduced mass of the α-α system. ε is the relative energy of the two clusters in the C.M. system. The direct Coulomb potential $V_C(r)$, the direct nuclear potential $V_D(r)$ and the kernel $K_l(r, r')$ are given by the Resonating Group Method' and depend upon the explicit form of the nucleon-nucleon potential which appears in the eight-particle Hamiltonian operator, H. We should notice that l is the relative angular momentum of the two a-clusters. The non-locality of the total potential of Eq. (3) arises from the possibility of nucleon exchanges. We see that the Resonating Group Method shows that the α-α interaction i) is non-local; ii) is angular momentum state dependent (this is a consequence of (i)); iii) is energy dependent (this dependence is not very strong).

By solving Eq. (3) with the proper boundary conditions, scattering phase-shift values can be obtained and, consequently, differential cross-sections. Clearly, constructing solutions for an equation like Eq. (3) is a difficult task. Therefore it is legitimate to ask ourselves if it is possible to construct an *effective local potential*, $V_{l,\varepsilon}(r)$, which hopefully depends only weakly on ε and l, and for which[2,3,4]

$$V_{l,\varepsilon}(r) \simeq V_c(r) + V_D(r) + \varphi_{l,\varepsilon}(r)^{-1}\int_0^\infty K_l(r, r')\,\varphi_{l,\varepsilon}(r')\,dr'. \tag{5}$$

By construction, $V_{l,e}(r)$, should have the same physical properties as that (non-local) calculated from the Resonating Group Method. The effective a-a interaction should have the follawing properties (which are necessary if the a-particle model is to give a satisfactory description of ^{8}Be):

i) The a-a interaction must become strongly repulsive at α-α spacings of less than 3 fm. This prevents the a's from overlapping and consequently sustains the a-particles'identities in ^{8}Be. This is supposed to simulate the Exclusion Principle for the fermions which make up the a-particles;

ii) To sustain the nucleus against electrostatic repulsion, the a-a forces must be attractive over spacing of 3-4 fm (this corresponds to the average spacing of two a's in a nucleus').

In this paper three different kinds of local phenomenological α-α potentials are studied. The three forms are (a) Eckart[5,6] – type potential; (b) hard-core square well and (c) soft-core square well potentials. In fact, the main idea of this paper was to study only the Eckart potential, because it has appeared in some recent papers[6] but no formal study has been done previously to determine how good it is as an a-a potential. The other two a-r: potentials were considered because of their simple mathematical shapes which helped to check the more complicated computer program for the Eckart potential. But from the study of the above potentials we learned some facts that were not quite in agreement with previous works[3,7].

Each of the potentials contain a few essential parameters which are fitted to a-a scattering data. We preferred to fit the differential cross-sections instead of the usual nuclear phase-shifts(actually the differential cross-section is the only *measured* quantity). In Sections (2), (3) and (4) those mentioned potentials are studied and the results discussed; in Section (5) we give a conclusion. An Appendix is added to give the mathematical method used here.

2. Hard-Core Square Well Potential-Calculations and Results

The effective potential, $V_{\alpha\alpha}(r)$, is defined to be[8]

$$V_{\alpha\alpha}(r) = \begin{cases} \infty, & r \leq a, \\ -V_a, & a < r \leq b, \\ V_c(r), & r \geq b, \end{cases} \tag{6}$$

where V_α is the attractive part, and a and b are the inner and outer radii respectively. V_c is the Coulomb potential and V_a and a are free parameters to be determined from a least-squares fit of the differential cross-section $d\sigma/d\Omega$, as described in the Appendix. It is obvious that one could rewrite $V_{\alpha\alpha}(r)$ as[8]

$$V_{\alpha\alpha}(r) = V_{\alpha\alpha}^{(N)}(r) + V_c(r), \tag{6'}$$

where

$$V^{(N)}(r) = \begin{cases} \infty, & r \leq a, \\ -V_a, & a < r \leq b, \end{cases} \tag{6''}$$

and

$$V_c(r) = (4e^2/r) \times \theta(r - b), \tag{6'''}$$

in agreement with the notation used in the Appendix. The parameter b was fixed at 4.1 fm. (b is expected to be weakly l- and E- dependent').

Potentials of this form were considered previously by Kermode[3], and by Van der Spuy[7].

By matching the wavefunction inside the nuclear domain with the one outside, one finds the nuclear phase shifts $v_l(E)$ and subsequently $d\sigma/d\Omega$. The matching point was taken here to be r = b. The data for the nuclear phase shifts and differential cross-sections at energies less than 20 MeV (laboratory system) were taken from references 9-13. We found that only the $l = 0, 2, 4$ partia1 waves were required to reproduce the experimental cross-sections. The parameter values obtained were

$$a = 2.17 \text{ fm} \qquad \text{and} \qquad V_a = 11.46 \text{ MeV}.$$

Defining the variance of the residuals as the unbiased quantity

$$\chi^2 = \sum_i \left| \left[\left(\frac{d\sigma}{d\Omega} \right)_{\text{th}} - \left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}} \right] \Big/ \sqrt{\left(\frac{d\sigma}{d\Omega} \right)_{\text{exp}}} \right|^2, \tag{7}$$

(unbiased since it treats all of the experimental points equally although please note that since it is not dimensionless it is not the usual χ^2), where σ_{th}'s are the theoretical differential cross-sections (our model) and σ_{exp}'s are the experimental ones, one finds $\chi^2 = 19.8$ mbarns/steradian (from now on let us represent the units of χ^2 by u). The parameter values above give a local effective α-α potential, approximately l and E independent

($E_{\alpha\alpha} < 40$ MeV) which reproduces the experimental cross-sections, as illustrated in Figs. (1) and (2) for some typical fits. It should be mentioned that an attempt was made to include in the data the resonance point[3]

$$E_{lab} = 184.24 \pm 0.1 \quad \text{keV} \qquad \text{for} \qquad l = 0,$$

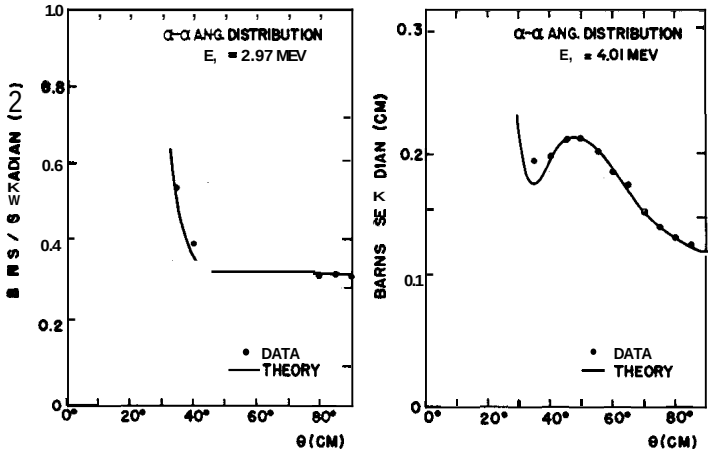

Fig. 1. a-a angular distributions at different bombarding energies. The absolute differential cross-section is in the C. M. scattering angle. $V_{\alpha\alpha}(r)$ is taken to be a hard-core square well potential.

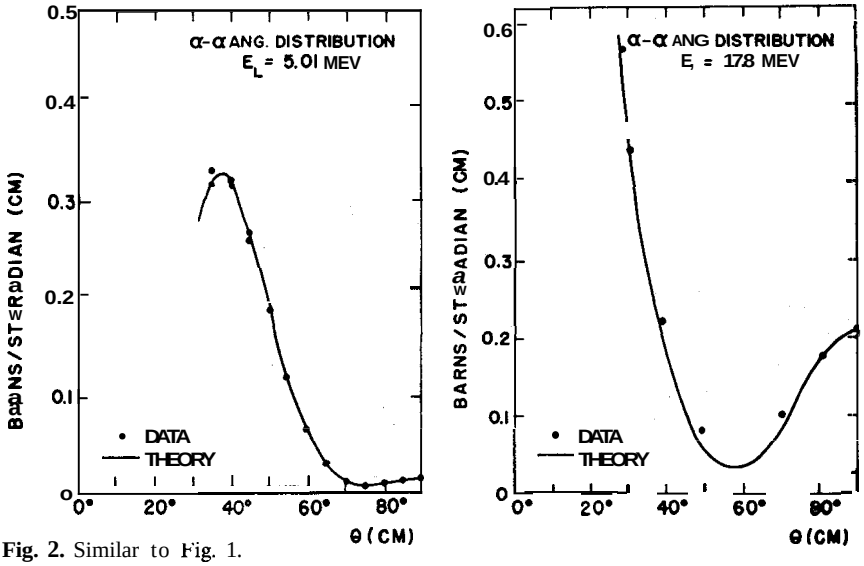

Fig. 2. Similar to Fig. 1.

(i.e. $\vartheta_c = 90°$), but we found it impossible in this model. Consequently, that point was deleted from the data set. One can expect to reproduce the experimental differential cross-sections for lab. energies less than *40* MeV. In the interval *40-50* MeV, there are some resonances in the experimental nuclear phase shifts which are not well understood and probably originate from nucleon exchange. Since we did not take into account the nucleon exchange term in our phenomenological potential, we did not expect to exactly reproduce the resonances with our method.

3. Soft-Core Square Well Potential. Calculations and Results

Define $V_{\alpha\alpha}(r)$ as

$$V_{\alpha\alpha}(r) = \begin{cases} V_r, & r \leq a, \\ V_a, & a \leq r \leq b, \\ V_c(r), & r \leq b, \end{cases} \tag{8}$$

where V_r, V_a and a are the free parameters to be determined by the least-squares fit. They represent the repulsive interaction, the attractive interaction and the inner radius, respectively, of $V_{\alpha\alpha}(r)$. Once again, b was taken to be fixed and equal to 4.1 fm and it will be the matching point. $V_c(r)$ is the Coulomb interaction. Similar analysis to the previous case was carried out here. No attempt was made to include the $l = 0$ resonance point at $E_{lab} = 184.24$ keV. The best set of parameter values found were

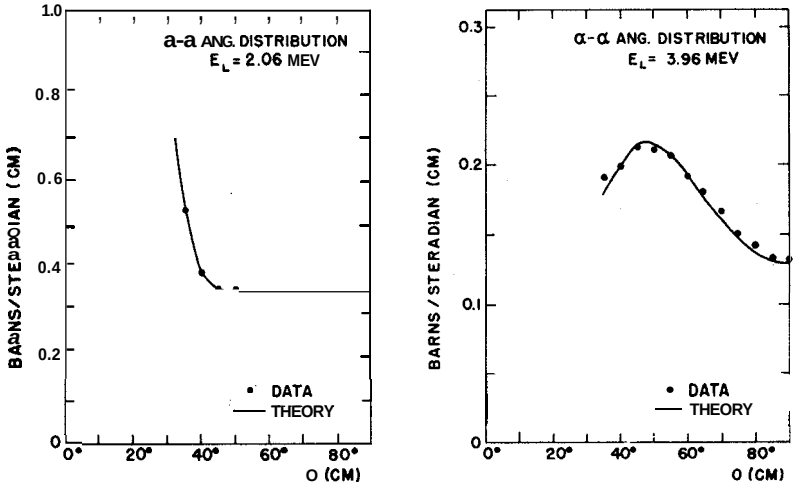

Fig. 3 - Similarly to Fig. 1, here $V_{\alpha\alpha}(r)$ is taken to be a soft-core square well potential.

$V_r = 18$ MeV, $V_a = 13.78$ MeV, a = 3.015 fm, with $\chi^2 = 17.5$ u. As is shown in Figs. (3) and (4), a fit of the experimental cross-section has been obtained and only 1 = 0, 2, 4 partial waves were required.

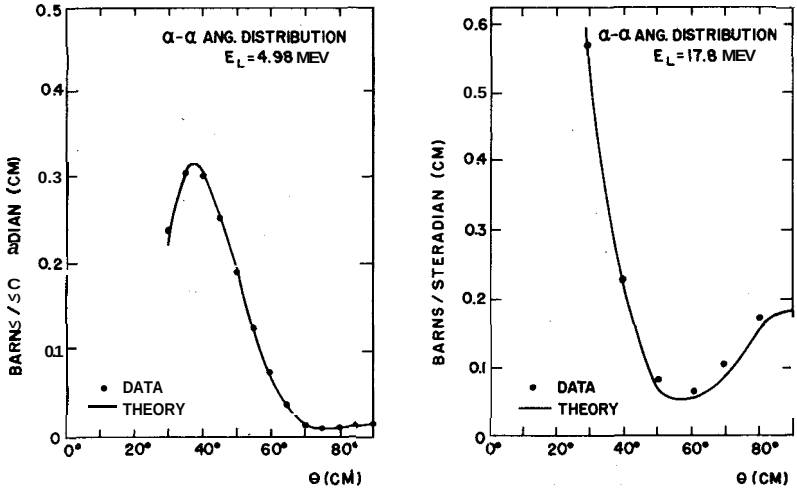

Fig. 4 - Similar to Fig. 3.

4. Eckart-Type Potenfial. Calculations and Results

The Eckart potential has the functional form

$$V_{aa}^{(N)}(\mathrm{r}) = \left(\frac{\hbar^2}{2\mu R^2}\right) \times \frac{\mathrm{n}(\mathrm{n}+1)}{(e^{r/R}-1)^2} - \frac{V_0}{e^{r/R}-1}, \tag{9}$$

where μ is the reduced mass, and n, R and V_0 are the adjustable parameters. This potential simulates the exclusion principle, which is one of our requirements and,, besides, if we use it in the 1= 0 effective two-body Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V_{\alpha\alpha}(r)\right]\varphi_0(r) = E\varphi_0(r), \tag{10}$$

it gives for $\varphi_0(r)$ the function

$$\varphi_0(r) = (1 - e^{-r/R})^{n+1}\, e^{-K_0 r}, \tag{11}$$

which has been successfully used by Noble[6,13,14], and by Coelho[11] to represent single particle a-cluster wavefunctions in ^{16}O. K_0 is related to the a-a binding energy (in the absence of Coulomb repulsion) by

$$K_0 = \frac{2\mu}{\hbar^2} B, \qquad \text{where} \qquad B = 2.63 \text{ MeV (Ref. 1)}.$$

Taking for n the same value $(n = 4)$ as that used in the paper by Noble and Coelho[13], one has R and V_0 as free parameters. Since the fitting procedure is non-linear, it is very important to start with a reasonable guess for R and V_0. Substitution of *Eq. (11)* into **Eq.** *(10)* gives the following relation between the parameters:

$$V_0 = \frac{(n+1)\hbar^2}{2\mu R^2}(1 + K_0 R). \tag{12}$$

R is expected to be of the order of *2* fm, so that $V_0 \approx 25$ MeV

The $V_{\alpha\alpha}(r)$ potential here is defined as

$$V_{\alpha\alpha}(r) = V_{\alpha\alpha}^{(N)}(r) + V_c(r). \tag{13}$$

Using the same set of data and proceeding exactly as in the two previous cases, onde finds the following set of parameters $R = 1.489$ fm, $V_0 = 52.78$ MeV. χ^2 turned out to be large, $\chi^2 = 837$ u. As already mentioned, the fitting procedure is non-linear and, consequently, possible different sets of starting values of the parameters should be analysed to see whether we have found a local minimum. However, after several trials it was concluded that the set found above is probably the best one. We could not obtain good fits of the cross-section. We conclude that the Eckart-type potential does not represent very well the effective a-a potential. It should be mentioned, however, that we could fit the $l = 0$ experimental nuclear phase shift v, very well with $R = 2.479$ fm and $V_0 = 23.05$ MeV. It is worthwhile also to mention that attempts were made to consider $V_{\alpha\alpha}^{(N)}(r)$ as

$$V_{\alpha\alpha}^{(N)}(r) = \left(\frac{\hbar^2}{2\mu R_1^2}\right) \frac{n(n+1)}{[\exp(r/R_1) - 1]^2} - \frac{V_0}{e^{r/R_2} - 1}, \tag{14}$$

where $R_1 \neq R_2$. However, we always obtained the best fits for $R_1 = R_2$. Fig. *5* shows some fits to the experimental cross-section.

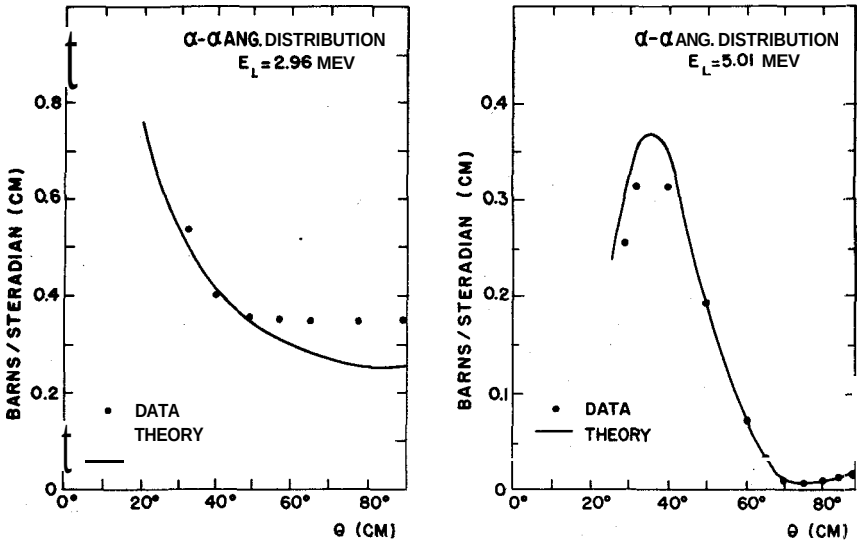

Fig. 5 - Similarly to Fig. 1, here $V_{\alpha\alpha}(r)$ is taken to be an Eckart potential

5. Conclusion

Considerations of nucleon-wavefunction antisymmetry (the Resonating Group Method) imply that acceptable a-a potentials must simulate the Exclusion Principle, must become strongly repulsive at α-α spacings of less than 3 fm and must be attractive over a-a spacings of 3-4 fm. The reason for constructing a local, effective $V_{\alpha\alpha}(r)$ is to simplify calculations in contrast with the corresponding calculations using the non-local potentials of the Resonating Group Method. The goal here was to construct potentials which are approximately 1-and E- independent (at least for low energies), and are able to reproduce the experimental cross-sections.

The first a-a potential considered was the Hard-Core Square Well and the set of parameter values found reproduced the experimental cross-sections for $E_{\alpha\alpha} < 40$ MeV. The Hard-Core Square Well potential has also been considered by Kermode[3]. However, his parameter values a r 1.7 fm., $V_a = 5.422$ MeV are different from the ones found here. The only reasonable explanation is that he used an effective range analysis, which is not as precise as the method used here.

The next potential analysed was the Soft-Core Square Well, which has

been used in some K-harmonics[16,17] calculations and in a paper on forward a-particle scattering by ^{16}O by Coelho''. The set of parameters found reproduced the experimental cross-sections for. $E_{lab} < 40$ MeV.

The last potential examined had the Eckart[5,6] form. It was disappointing to find that this potential was unable to give a good fit of the experimental cross-section data, except for the s-wave nuclear phase shift. It is clear then that the Eckart potential is applicable only in the case of low energy.

Only *s*, *d* and g partial waves needed to be included in our analysis. As could be seen in Fig. (6), for low energies ($E_{lab} < 40$ MeV), higher partial waves do not give any important contribution to the cross-section. To see how strongly *l*-dependent a given $V_{\alpha\alpha}(r)$ is, we made a comparison of parameter

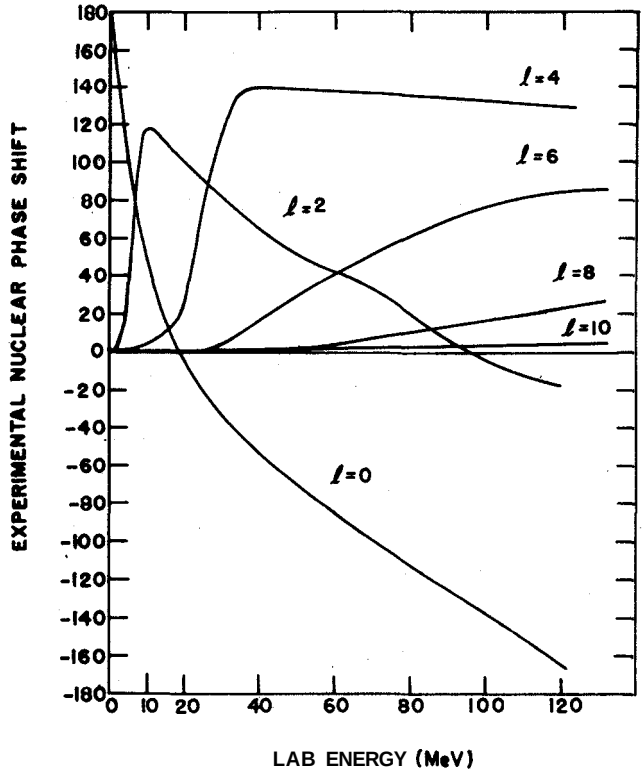

Fig. 6 - Experimental nuclear phase-shift versus laboratory energy of the bombarding α-particle. The s-wave nuclear phase-shift could very well be reproduced for ali three α-α potentials, for different sets of parameter values (for $E_{lab} < 40$ MeV).

values obtained from s-wave nuclear phase shift fitting with those obtained from a cross-section fit which includes s, *d* and g partial waves. The results for our potentials are summarized in the table. It is interesting to point out that for all three potentials, the s-wave nuclear phase shift can be very well fitted with more than one set of parameter values. This is a consequence of the non-linear nature of the fitting procedure. For instance, for the hard-core square well, the $1 = 0$ parameter values $a = 1.4$ fm., $b = 4.1$ fm. and $V_a = 5.422$ MeV, also obtained by Kermod[3], give a very good fit of the s-wave nuclear phase shift. For the soft-core square well, the range of variation of the parameters in the case of $l = 0$ is much wider, with V_r varying from 100 to 10 MeV, b kept fixed at 4.1 fm., and V_a and a approximately stable. To the extent that we add more partial waves, the location of an absolute minimum for χ^2 becomes more visible and restricted. We may say that for low energies, we could obtain a set of parameter values, approximately *l*-and E- independent, for hard-core and soft-core square well potentials which reproduces the α-α differential cross-sections. We finally should say that in passing from soft-core to hard-core square well potentials, the attractive part, V_a, varied from 13.78 to 11.46 MeV and the inner radius, a, from 3.015 to 2.170 fm in a continuous way (b was kept fixed at 4.1 fm).

Table 1 below summarizes all the results obtained.

Potential Type	Parameter Values				Partial waves	Reference
	a(fm)	*b*(fm)	V_a(MeV)			
Hard-Core	1.7	4.0	7.2		0	(1)
	1.4	4.1	5.422		0	(2)
	1.45	4.1	5.45		0	(3)
	1.7-1.9	4.1	5.422		0, 2, 4	(2)
	2.17	4.1	11.46		0, 2, 4	(3)
	a(fm)	*b*(fm)	V_a(MeV)	V_r(MeV)		
Soft-Core	3.015	4.1	13.78	18.0	0, 2, 4	(3)
	n		V_0(MeV)	*R*(fm)		
Eckart	4		23.05	2.479	0	(3)
	4		52.78	1.489	0, 2, 4	(3)

Table 1 - Parameter values obtained for the three effective potentials compared with previous works [(1) - Van der Spuy[7]; (2) - Kermode[3]; (3) - This work.].

Certainly, we may also try to take into consideration possible polarization and nucleon exchange effects. We may consider this in the future.

A word should be said about the numerical method utilized to handle Eq. (15) which appears in the Appendix. We have used Hamming's modified predictor-corrector method with the help of a fourth order Runge-Kutta method for the starting values. The solution was carried out on the IBM 360/65 computer of the University of Pennsylvania.

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Appendix

Let us denote by $V_{\alpha\alpha}(r)$ the effective a-a potential. The partial-wave Schrodinger equation for scattering wavefunctions can be written as

$$y''_{k,l}(r) + \left[k^2 - v_n(r) - v_c(r) - \frac{\quad}{r^2} \quad v_{k,l}(r) = 0, \right. \tag{15}$$

where μ is the reduced mass, and

$$k^2 = \frac{2\mu}{\hbar^2}E, v_n(r) = \frac{2\mu}{\hbar^2}V_n^{(N)}(r)$$

and

$$v_c(r) = \frac{2\mu}{\hbar^2}V_c(r),$$

corresponds to the Coulomb interaction, $V_c(r)$, given by

$$V_c(r) = \frac{Z_1 Z_2 e^2}{r} = \frac{4e^2}{r}.$$

$V_{\alpha\alpha}^{(N)}$ should be a short range potential $(V_{\alpha\alpha}(r) = V_{\alpha\alpha}^{(N)}(r) + V_c(r))$.

Let r, be the range of $V_{\alpha\alpha}^{(N)}(r)$, such that $V_{\alpha\alpha}^{(N)}(r) \overrightarrow{r \to r_n} 0$. Three intervals for the range of r are to be distinguished in Eq. (14):

1) $r < r$, : $V_{\alpha\alpha}^{(N)}(r)$ and $V_c(r)$ are important.

2) r, $< r <$ R: In this region, $V_{\alpha\alpha}^{(N)}(r) \simeq 0$ (R is assumed to be a very large number).

Thus, for r > R, Eq. (15) reduces to the Coulomb equation

$$y''_l(\rho) + \left[1 - \frac{2\eta}{\rho} - \frac{l(l+1)}{\rho^2}\right] y_l(\rho) = 0, \tag{16}$$

where $p = kr$ and $\eta = 4\mu e^2/\hbar^2 k$ (η is dimensionless).

Equation (16) has the solution[18]

$$y_l(\rho) = A(F_l(\rho) + \tan \nu_l(E)\, G_l(\rho)), \tag{17}$$

where $F_l(\rho)$ and $G_l(\rho)$ are the regular and irregular Coulomb functions, respectively, and $\nu_l(E)$ is the "nuclear" phase shift. Because of the interference effects between the Coulomb and nuclear potentials, the "nuclear" phase shift ν_l is not equal to that phase ν'_l which would be produced by the same nuclear potential in the absence of the Coulomb interaction.

3) $r > \mathbb{R}$: In this region we have the asymptotic solutions of **Eq.** (16).

The natural object to compare with experiment is the differential cross-section; $d\sigma/d\Omega$. In order to determine $d\sigma/d\Omega$ one must find $\tan\nu_l(E)$. By matching the internal ($r < r_i$) and external ($r \gg r_n$) logarithmic derivatives of the wave-functions at a point $b > r$, (where $V_{\alpha\alpha}^{(n)}(r)$ is negligible within a predetermined error $\varepsilon > 0$) we easily find that

$$\tan \nu_l(E) = -\left[\frac{F_l(\rho)\, y'_{k,l}(r) - kF'_l(\rho)\, y_{k,l}(r)}{G_l(\rho)\, y'_{k,l}(r) - kG'_l(\rho)\, y_{k,l}(r)}\right]_{r=b} \tag{18}$$

where

$$y'_{l,k}(r) = dy_{k,l}(r)/dr \qquad \text{and} \qquad F'_l(\rho) = dF_l(\rho)/d\rho.$$

As a-particles are bosons, the nuclear scattering amplitude for the reaction should be fully symmetrized, and is given by

$$f^{sym}(k, \theta) = f(k, 0) + f(k, \pi - \theta).$$

The quantity f is defined as[18] $f = f_n + f_c$, where f_n is the nuclear amplitude and f_c the Coulomb amplitude. Explicit expressions are

$$f_n(k, \theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l + 1)\, P_l(\cos \theta)\, e^{2i\sigma_l(E)}\, e^{i\nu_l(E)} \sin \nu_l(E), \tag{19}$$

where $\sigma_l(E)$ is the Coulomb phase shift, θ is the C.M scattering angle and

$$f_c = B_c \exp\left[2i\sigma_0(E) - 2iq \log \sin \theta/2\right] \tag{20}$$

B_c comes from the Coulomb Born Approximation and is given by

$$B_c = -\frac{\eta}{2k \sin^2 \theta/2}.$$

Symmetrizing f_n and f_c, only even values of l will contribute for the total amplitude.

Finally, the differential cross-section is given by

$$\frac{d\sigma(\theta, E)}{d\Omega} = |f^{sym}(k, \theta)|^2.$$

The parameters appearing in $V_{\alpha\alpha}(r)$ can be obtained through a non-linear least-squares fit to the differential cross-section $d\sigma/d\Omega$. In order to do this, we need to know $\tan\nu_l$ from Eq. (17). This requires the determination of the Coulomb functions, and $y_{k,l}(r)$ (and $y'_{k,l}(r)$). To obtain $y_{k,l}(r)$ and $y'_{k,l}(r)$, one must solve Eq. (15) numerically.

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