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Plasma Effects in Sound Amplification in Piezo-electric Semiconductors

L. C. M. MIRANDA and D. ter HAAR
*Department of Theoretical Physics, University of Oxford**

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By considering the influence of phonon emission by charge carriers moving in the field of plasma oscillations, we show that for piezo-electric semiconductors such as InSb and GaAs the sound wave amplification process may be appreciably larger than would follow from a theory in which plasma effects are neglected.

Considerando a influência da emissão de fonons por transportadores de cargas em movimento em um meio onde há oscilações de plasma, mostra-se que em materiais semicondutores piezo-elétricos, tais como InSb e GaAs, o processo de amplificação da onda sonora pode ser sensivelmente maior na presença de efeitos do plasma do que quando tais efeitos são ignorados.

1. Introduction

Hutson, McFee, and White[1] observed in 1961 that if an ultrasonic wave was incident upon a piezo-electric semiconductor such as CdS in which a current is flowing, the wave was amplified under certain conditions. This phenomenon has been explained as an example of phonon instability, another example of which is the occurrence of current saturation in the steady state. Pustovoit[2], McFee[3], and Spector[4] have written review articles describing various aspects both of the experimental and of the theoretical work in the field.

Qualitatively the occurrence of sound amplification, which occurs when the drift velocity of the electrons v_d exceeds the sound velocity v_s, can be explained as follows[5,6,7,8]. The sound wave produces a redistribution of the carriers in space, that is, it produces a space charge density distribution. The applied electric field will cause the space charge to drift; if the drift velocity is supersonic, the bunches of carriers will emit phonons

*Postal Address: 12 Parks Road, Oxford OX1 3PQ.

– in complete analogy with the Cherenkov effect. This amplification mechanism will work only if the condition

$$kl \ll 1 \tag{1}$$

is satisfied, where k is the sound wave number and l the carrier mean free path. Condition (1) means that the sound frequency is much smaller than the carrier collision frequency so that the carriers undergo many collisions while traveling a distance equal to the sound wavelength which entails, in turn, that the drift velocity is the only velocity to be compared with v_s. If, however,

$$kl \gg 1, \tag{2}$$

single carriers can emit Cherenkov phonons, as the thermal velocity (or Fermi velocity) normally comfortably exceeds the sound velocity. Even in the absence of an external field, the carriers can in that case be in resonance with the wave field, that is, move with the same velocity and thus remain in phase and resonantly lose or gain energy to the wave[8,9].

In order to evaluate the amplification – or negative damping – we need to know the conductivity tensor. In a semi-classical approach, one evaluates this tensor using the free-electron gas model for the conduction electrons and the Boltzmann equation to determine the electron distribution function in the presence of an acoustic wave[4,10]. In the original theory, the current was determined by the electrical conductivity and the diffusion equation, which excludes automatically the possibility to take into account plasma oscillations of the current carriers. Recently, however, Kornyushin[11], using a hydrodynamical description of the current carriers, has shown that plasma oscillations may affect the sound amplification, provided the plasma frequency ω_p is larger than the reciprocal of the electron relaxation time for scattering by phonons, τ, or

$$\omega_p \tau > 1. \tag{3}$$

The plasma frequency is given by the relation

$$\omega_p^2 = \frac{4\pi n_0 e^2}{m\varepsilon}, \tag{4}$$

where n_0 is the carrier density, e their charge, m their mass, and ε the dielectric constant. The relaxation time τ is related to the mobility μ by the equation

$$\mu = e\tau/m. \tag{5}$$

Physically, one can see that, if condition (3) is satisfied, plasma oscillations may be important as follows. This condition means that we are essentially dealing with a collisionless plasma and such a system can – as is well known – sustain collective oscillations with frequencies ω satisfying the dispersion law

$$\omega = \omega_p + (\mathbf{k} \cdot \mathbf{v}_d) \tag{6}$$

where k is the wavevector of the collective modes.

If, on the other hand, we have a situation where

$$\omega_p \tau \ll 1, \tag{7}$$

plasma oscillations will be strongly damped and the original calculations remain valid.

There is, we feel, another argument in favour of including plasma effects based on an estimate of the ratio of the plasmon energy density, W_{pl}, to the particle kinetic energy density, W_{part}. If k_D is the Debye wavevector,

$$k_D = \left(\frac{4\pi n_0 e^2}{\varepsilon k_B T}\right)^{1/2}, \tag{8}$$

where k_B and T are, respectively, the Boltzmann constant and the absolute temperature, we have

$$W_{pl}/W_{part} \approx (k_B T k_D^3)/(n_0 k_B T). \tag{9}$$

If one takes the case of InSb for the case where $\omega_p \tau > 1$ and $kl > 1$, we have[12] $n_0 \approx 4.10^{14}\,\mathrm{cm}^{-3}$, $\varepsilon \approx 18$, and hence $W_{pl}/W_{part} \approx 0.1$, which indicates that plasma effects could be not entirely negligible.

In Section 2, we shall find the contribution from plasmons to the sound damping coefficient; this quantity is evaluated in Section 3, and compared with the damping coefficient calculated by Spector[4] in Section 4 for the case of InSb and GaAs.

2. Formulation of the Problem

We saw in the introduction that the sound amplification is essentially determined by the electron-phonon interaction which is also involved in the phenomenon of current saturation[13,14,15] and in the derivation of the dispersion relation for the acoustic waves[16]. In what follows, we shall restrict our discussion to piezo-electric semiconductors such as InSb or GaAs and we shall consider only longitudinal acoustic phonons – one can easily include the transverse phonons.

In InSb and GaAs, the main interaction between the acoustic phonons and the current carriers is the piezo-electric coupling[17], while for frequencies up to $\omega \sim 10^{13}$ Hz the deformation potential coupling[4,18] can be shown[4] to be negligible. Moreover, one can also neglect phonon-drag[19] effects, as the ratio of the induced electric fields caused by phonon-drag and by piezo-electric coupling, respectively, is of the order of 10^{-5}. The coupling constant $V_{\mathbf{k}}$ between the carriers and the (longitudinal) piezo-electric acoustic phonons of frequency $\omega_{\mathbf{k}} = kv_s$ is given by the equation

$$V_{\mathbf{k}} = \left(\frac{\hbar C}{2\Omega\omega_{\mathbf{k}}}\right)^{1/2}, \tag{10}$$

where

$$C = \left(\frac{4\pi e\beta}{\varepsilon}\right)^2 \frac{1}{\rho}, \tag{11}$$

with Ω the volume of the system, ρ its mass density, and β the piezo-electric constant.

Let us now consider how plasmons can give up energy to the acoustic phonons. Except for very low temperatures below 1°K, and very low densities, the plasmon frequency branch and the acoustic phonon branch do not intersect[20], so that the direct interaction between these two branches is weak; one can neglect the (plasma) wave – (acoustic) wave interaction processes. However, there is the possibility of the process

$$\text{plasmon} + \text{charge carrier} \rightarrow \text{phonon} + \text{charge carrier}. \tag{12}$$

This process corresponds to the emission of an acoustic phonon (energy $\omega_{\mathbf{k}}$, wavevector k) by a charge carrier which oscillates in the field of a plasmon (energy $\omega_{\mathbf{k}'}$, given by equation (6), wavevector $\mathbf{k}'$); in the process the electron energy and momentum change, respectively, by $\omega_{\mathbf{k}'} - \omega_{\mathbf{k}}$ and $\hbar(\mathbf{k}' - \mathbf{k})$. This process is analogous to the non-linear Landau damping of plasma waves in plasma physics[21].

The kinetic equation for the distribution function $N_{\mathbf{k}}$ of the acoustic phonons is of the form

$$\begin{aligned}\frac{dN_{\mathbf{k}}}{dt} = \sum_{\mathbf{p},\mathbf{k}'} \frac{2\pi}{\hbar} |M|^2 \{ & f_{\mathbf{p}} \tilde{N}_{\mathbf{k}'} (N_{\mathbf{k}} + 1)(1 - f_{\mathbf{p}+\mathbf{k}'-\mathbf{k}}) \\ & - f_{\mathbf{p}+\mathbf{k}'-\mathbf{k}} N_{\mathbf{k}} (\tilde{N}_{\mathbf{k}'} + 1)(1 - f_{\mathbf{p}})\} \delta(\varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{k}'} - \varepsilon_{\mathbf{p}+\mathbf{k}'-\mathbf{k}} - \hbar\omega_{\mathbf{k}}),\end{aligned} \tag{13}$$

where $|M|^2$ is the square of the matrix element of the process (12), $f_{\mathbf{p}}$ is the electron (carrier) distribution function, $\tilde{N}_{\mathbf{k}'}$ the plasmon distribution function, and $\varepsilon_{\mathbf{p}} = \hbar^2 p^2/2m$ the electron energy. Using the normal many-body techniques[21,22], the matrix element is given by the equation

$$M = M(\mathbf{k}')V(\mathbf{k})\left[\frac{1}{\varepsilon_{\mathbf{p}} + \hbar\omega_{\mathbf{k}'} - \varepsilon_{\mathbf{p}+\mathbf{k}'}} + \frac{1}{\varepsilon_{\mathbf{p}+\mathbf{k}'-\mathbf{k}} - \varepsilon_{\mathbf{p}-\mathbf{k}} - \hbar\omega_{\mathbf{k}'}}\right], \tag{14}$$

where $V(\mathbf{k})$ is given by equation (10) and $M(\mathbf{k}')$ is the electron-plasmon vertex given by[23]

$$M(\mathbf{k}') = \left[\frac{2\pi e^2 \hbar\omega_{\mathbf{k}'}}{\Omega k'^2}\right]^{1/2}. \tag{15}$$

To get the equation of motion for the sound wave intensity $I_{\mathbf{k}} (= N_{\mathbf{k}}\hbar\omega_{\mathbf{k}})$ we multiply equation (13) by $\hbar\omega_{\mathbf{k}}$, transform the summations to integrals and take the classical limit, that is, retain the lowest-order terms in $\hbar$. We then get

$$\frac{dI_{\mathbf{k}}}{dt} = \gamma I_{\mathbf{k}} + \gamma', \tag{16}$$

where

$$\gamma = \gamma_1 - \gamma_2, \tag{17}$$

with

$$\gamma_1 = -\frac{\Omega^2}{(2\pi)^3}\int d^3v \int d^3k' \frac{2\pi}{\hbar^2}\,\frac{M^2(\mathbf{k}')V^2(\mathbf{k})}{m^2}\,\frac{\hbar}{m}\left([\mathbf{k}'-\mathbf{k}]\cdot\frac{\partial f}{\partial \mathbf{v}}\right)$$
$$\times\ \tilde{N}(\mathbf{k}')\,\delta[\omega_{\mathbf{k}'} - \omega_{\mathbf{k}} - (\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})], \tag{18}$$

$$\gamma_2 = \frac{\Omega^2}{(2\pi)^3}\int d^3v \int d^3k' \frac{2\pi}{\hbar^2}\,\frac{M^2(\mathbf{k}')V^2(\mathbf{k})}{m^2}\,\frac{(\mathbf{k}\cdot\mathbf{k}')}{[\omega_{\mathbf{k}'} - (\mathbf{v}\cdot\mathbf{k}')]^4}\,f(\mathbf{v})$$
$$\times\ \delta[\omega_{\mathbf{k}'} - \omega_{\mathbf{k}} - (\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})], \tag{19}$$

$$\gamma' = \frac{\Omega^2}{(2\pi)^3}\int d^3v \int d^3k' \frac{2\pi}{\hbar^2}\,\frac{M^2(\mathbf{k}')V^2(\mathbf{k})}{m^2}\,\frac{(\mathbf{k}\cdot\mathbf{k}')}{[\omega_{\mathbf{k}'} - (\mathbf{v}\cdot\mathbf{k}')]^4}$$
$$\times\ \hbar\omega_{\mathbf{k}}\tilde{N}(\mathbf{k}')\,f(\mathbf{v})\,\delta[\omega_{\mathbf{k}'} - \omega_{\mathbf{k}} - (\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})], \tag{20}$$

where we have put $\hbar\mathbf{p}/m = \mathbf{v}$, $\sum_{\mathbf{p}}(\ldots) \to \Omega \int d^3p(\ldots)$, $\sum_{\mathbf{k}}(\ldots) \to \Omega(2\pi)^{-3} \int d^3\mathrm{k}(\ldots)$, and assunied that $f_{\mathbf{p}} \ll 1$, $N_{\mathbf{k}} \gg 1$, but $\hbar\omega_{\mathbf{k}} N_{\mathbf{k}} = I$, = finite; this is justified as the semiconductors under the experimental conditions considered will be non-degenerate. The experimentally produced drift velocities are less than or of the order of magnitude of the thermal velocities of the carriers.

If the quantity γ in equation (16) is positive (negative), we get amplification (damping) of the sound waves. The spontaneous emission term γ' is not important for our problem.

3. Evaluation of the Damping Coefficient

Substituting $M(\mathbf{k}')$ and $V(\mathbf{k})$ from equations **(10)** and **(15)** and using for $f(\mathbf{v})$ the displaced Maxwell-Boltzmann distribution function,

$$f(\mathbf{v}) = \frac{n_0}{(2a)^{3/2}} \exp\left[-(\mathbf{v}-\mathbf{v}_d)^2/a\right], \tag{21}$$

with

$$a = \frac{2k_B T}{m}, \tag{22}$$

we get

$$\gamma_1 = A \int d^3v \int d^3k' \frac{\hbar\omega_{\mathbf{k}'} \cos^2\theta}{k_B T} \frac{(\{\mathbf{k}'-\mathbf{k}\} \cdot \{\mathbf{v}-\mathbf{v}_d\})}{[\omega_{\mathbf{k}'} - (\mathbf{v}\cdot\mathbf{k}')]^4} \exp\left[-(\mathbf{v}-\mathbf{v}_d)^2/a\right] \times \tilde{N}(\mathbf{k}')\,\delta[\omega_{\mathbf{k}'} - \omega_{\mathbf{k}} - (\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})], \tag{23}$$

$$\gamma_2 = A \int d^3v \int d^3k' \frac{\omega_{\mathbf{k}'} \cos^2\theta}{[\omega_{\mathbf{k}} - (\mathbf{v}\cdot\mathbf{k}')]^4} \exp\left[-(\mathbf{v}-\mathbf{v}_d)^2/a\right] \delta[\omega_{\mathbf{k}'} - \omega_{\mathbf{k}} - (\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})], \tag{24}$$

where θ is the angle between k and **k'**, and

$$A = \frac{e^2\beta^2}{m\rho\varepsilon} \cdot \frac{k^2\omega_{\mathbf{p}}^2}{\omega_{\mathbf{k}}(2a)^{3/2}}. \tag{25}$$

We shall assume that the plasmons are at equilibrium, which is consistent with the use of expression *(21)* for $f(\mathbf{v})$. Moreover, for the conditions considered by us *tio,.* $\ll k_B T$, so that we have

$$\tilde{N}(\mathbf{k}') = [\exp(\hbar\omega_{\mathbf{k}'}/k_B T) - 1]^{-1} \approx \frac{k_B T}{\hbar\omega_{\mathbf{k}'}}. \tag{26}$$

We now get

$$\gamma_1 = A\int d^3v \int d^3k' \frac{\cos^2\theta(\{\mathbf{k}'-\mathbf{k}\}\cdot\{\mathbf{v}-\mathbf{v}_d\})}{[\omega_{\mathbf{k}'}-(\mathbf{v}\cdot\mathbf{k}')]^4} \exp[-(\mathbf{v}-\mathbf{v}_d)^2/a]\delta[\omega_{\mathbf{k}'}-\omega_{\mathbf{k}}-(\mathbf{v}\cdot\{\mathbf{k}'-\mathbf{k}\})]. \tag{27}$$

As plasmons are well-behaved excitations only when their phase velocity is larger than the thermal velocity and when their wavenumber is less than k_D, we can put $\omega_{\mathbf{k}'} - (v.k') \approx$ *o,,* and we get

$$\gamma_1 = A\frac{4\pi a}{\omega_p^4}[\omega_p - \omega_{\mathbf{k}} + (\mathbf{v}_d\cdot\mathbf{k})]\int_0^{k_D} k'^2\,dk' \int_0^{\pi} \frac{\sin\theta\cos^2\theta\,d\theta}{[k'^2 + k^2 - 2k'k\cos\theta]} \tag{28}$$

and similarly

$$\gamma_2 = \frac{4\pi a A}{\omega_p^3}\int_0^{k_D} k'^2\,dk' \int_0^{\pi} \frac{\sin\theta\cos^2\theta\,d\theta}{[k'^2 + k^2 - 2k'k\cos\theta]^{1/2}}, \tag{29}$$

and hence, from equation (17),

$$\gamma = \frac{k(v_d - v_s)}{\omega_p}\gamma_2, \tag{30}$$

if we assume, for the sake of simplicity, that $\mathbf{v}_d$ is parallel to $\mathbf{k}$.

Evaluating the integrals in equation (29), by expanding the denominator in terms of Legendre polynomials and distinguishing between the cases where $k_D/k = \eta < 1$ or $\eta > 1$, we find finally,

$$\gamma = \frac{\varepsilon C}{2000\pi m(2a)^{1/2}}\cdot\frac{k_D^5}{\omega_p^2\omega_{\mathbf{k}}}\cdot\frac{50 + 12\eta^2}{\eta^2}(v_d - v_s), \quad \eta \leq 1; \tag{31a}$$

$$\gamma = \frac{\varepsilon C}{2000\pi m(2a)^{1/2}}\cdot\frac{k_D^5}{\omega_p^2\omega_{\mathbf{k}}}\cdot\frac{75(\eta^2 - 1) + 62 + 60\ln\eta}{\eta^5}(v_d - v_s), \quad \eta \geq 1. \tag{31b}$$

If we use equation (8), write

$$x = v_d/v_s, \qquad v_{th} = (k_B T/m)^{1/2},$$

and retain only the leading terms in equations (31), we get

$$\gamma = \frac{\beta^2 \omega_p^2 k^2}{15 \rho n_0 v_s^3}(x-1), \qquad k \le k_D; \tag{32}$$

$$\gamma = \frac{\beta^2 \omega_p^3 k}{22 \rho n_0 v_{th}^4}(x-1), \qquad k > k_D \tag{33}$$

Hence, as we expected, amplification occurs when $v_d > v_s$ and damping when $v_d < v_s$.

The case $k > k_D$ is unlikely to occur, as the maximum momentum transferred from the plasmons to the phonons is of the order $\hbar k_D$. We shall, therefore, restrict our discussion to the case $k \le k_D$.

4. Discussion

Let γ_{pl} be the contribution to the damping to the plasmon-phonon process (12), which is given by equation (32). We must compare this result with the result obtained by Spector[4] when plasma effects are neglected. Spector's results depend on whether $kl < 1$ or $kl > 1$ (see the discussion in the Introduction) and are given by the equation

$$\gamma_{Sp} = \frac{\beta^2 \omega_F \tau}{3 \rho v_s^2} \frac{x-1}{(x-1)^2 + (k_c/k)^2[1 + (k/k_D)^2]^2}, \quad kl < 1; \tag{34}$$

$$\gamma_{Sp} = \frac{\beta^2 v_{th}}{\rho \omega_p^2} \frac{k^2}{[1 + (k/k_D)^2]^2}(x-1), \qquad kl > 1, \tag{35}$$

where τ is again the electron relaxation time, and

$$l = v_{th}\tau, \qquad k_c = \omega_p^2 \tau / v_s. \tag{36}$$

For the ratio $\lambda = \gamma_p/\gamma_{Sp}$, we thus have

$$\lambda(kl < 1;\ k \le k_D) = \frac{k^2 v_s^2}{5 n_0 \tau v_F}\left\{(x-1)^2 + \left(\frac{k_c}{k}\right)^2\left[1 + \left(\frac{k}{k_D}\right)^2\right]^2\right\}, \tag{37}$$

$$\lambda(kl > 1;\ k \le k_D) = \frac{k_D^4}{15 n_0 k}\left[s + \left(\frac{k}{k_D}\right)^2\right]. \tag{38}$$

For CdS, under the experimental conditions used[1,13,15], the condition $\omega_p \tau > 1$ is not satisfied, but it can be satisfied for InSb and GaAs. We shall use the values of the parameters given in Table 1, which are taken from Refs. 12, 24 and 25.

As we have considered acoustic-phonon scattering as the dominant electron scattering process[4], we must restrict our discussion to the temperature range 77 – 300°K when this assumption is justified. In that temperature range one may assume the electron mobility to be independent of the applied field[4].

Table **1**

	$n_0(\mathrm{cm}^{-3})$	$m(\mathrm{g})$	ε	$\tau(\mathrm{s})$	$l(\mathrm{cm})$	$k_D l$
InSb	4.10^{14}	2.10^{-29}	18	6.10^{-12}	1.4×10^{-4}	11
GaAs	2.10^{12}	7.10^{-29}	12	5.10^{-13}	6.10^{-6}	4.2

From Table 1, we see that $kl > 1$ for InSb and GaAs under the conditions considered and, as the plasma effect is largest when $k \sim k_D$, we consider $\lambda \equiv \lambda(kl > 1;\ k \sim k_D)$ for which from equation (38) we get

$$\lambda = \frac{4}{15}\left(\frac{4\pi n_0 e^2}{\varepsilon k_B T}\right)^{1/2}, \tag{39}$$

or

$$\lambda_{\mathrm{InSb}} \sim 0.3, \qquad \lambda_{\mathrm{GaAs}} \sim 3, \tag{40}$$

so that, indeed, plasma effects may well be significant in sound amplification effects in piezo-electric semiconductors.

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