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Sigle Particle States in Nuclei

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1. Introduction

During the last few years, many studies of nucleon transfer and knockout reactions have given much detailed information on the energies and spectroscopic factors of the single particle states of nuclei throughout the periodic table. At first sight, these data appear to relate only to the specific properties of individual nuclei, but more careful examination shows underlying systematic features that vary quite smoothly from one nucleus to the next.

The principal systematic behaviour concerns the mean energies of the single-particle states, and it has been found possible to express them as eigenvalues of a simple local potential whose depth depends smothly on the atomic weight and the nuclear symmetry parameter. This work is described in Sec. 2.

Such studies require accurate spectroswpic data and it becomes important to find ways of improving the current techniques. A useful method, suggested recently by Clement, concerns the use of J-dependent sum rules. These relate the spectroscopic factors of stripping and pickup reactions on the same nucleus, and this extra constraint enables the accuracy of spectroscopic factors to be significantly increased (Sec 3).

2. The Systematics of Bound Single-Particle States

In most nuclei, the single particle states are fragmented into several components by the residual interactions but it is possible to define the energy of the unperturbed state as the 'centre of gravity' of the fragments, each weighted by its spectroscopic factor. It is also possible to define the moments of the fragment distribution but the experimental data on these quantities are at present rather unreliable. The centroid energies are, however, relatively unaffected by the omission of one or two weak compo-

nents and by any uncertainties in the absolute normalisation of the spectroscopic factors, so it is useful to examine these for systematic behaviour.

The first detailed attempt to correlate these single-particle energies as eigenvalues of a single potential was made by Cohen (1965) and many investigations have subsequently been made. These are of two types, viz., ihose examining particular single-particle states in a range of nuclei and those concerned primarily with severa1 single-particle states in a particular nucleus. The present work is of the former type and this makes possible an examination of the state dependence of the potential.

In the first stage of this work, all the available data for nuclei in the range $35 \leq A \leq 65$ have been analysed. This range was chosen partly because of the wealth of the data and partly because these nuclei are relatively free from the complications of highly deformed nuclei, so that a spherically symmetrical potential can be used with a spin-orbit term of the Thomas form, viz.,

$$V(r) = Vf_1(r) + V_S\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{df_2(r)}{dr}\ \mathbf{L}.\boldsymbol{\sigma}, \qquad (2\text{-}1)$$

where the form factors $f_{1,2}(r) = [1 + \exp\{(r - R_{1,2})/a_{1,2}\}]^{-1}$, with $R_{1,2} = r_{1,2}A^{1/3}$. The radius and diffuseness parameters were fixed for the main calculations at $r_1 = 1.25$ fm, $a_1 = 0.65$ fm, $r_2 = 1.1$ fm, $a_2 = 0.65$ fm. These values have frequently been used in scattering analyses, and the somewhat smaller value of r_2 compared with r_1 is suggested by analyses of polarisation data. The conclusions of this work are insensitive to the precise values chosen for the form factor parameter.

The choice of a local potential requires some comment, as nuclear matter calculations indicate that the true potential is partly local and partly non-local. Several investigations (Wyatt *et* al; 1960; Meldner and Sussmann, 1963; Meldner, 1967) have indeed used a non-local potential, although it has been found that a purely non-local potential is inadequate for light nuclei (Grimm *et. al.,* 1971). To use a potential with both local and non-local components would introduce too many arbitrary parameters. so we use the equivalent local potential. This has the additional advantage of leaving open the energy dependence of the potential. Since the wavefunctions corresponding to equivalent local and non-local potentials are not the same, this must be taken into account when the potentials are used to calculate nucleon density distributions.

The potential depth V was shown by Cohen to depend on the nuclear symmetry parameter $(N-Z)/A$, and an energy dependence is expected by analogy with the scattering situation. However, when analysing a large body of data for many nuclei it is more convenient to study each particular state (NLJ) separately, and this has the additional advantage of leaving open the possible state dependence of the potential. It is then natural to allow the potential to depend on A, and this also absorbs any inadequacies in the parametrization of the nuclear radius parameter R as a function of A (Hodgson, *1970).* Unlike the scattering phase shifts, the energy of a bound state is fixed by $V(r)$ so that V cannot without redundancy be allowed to depend on the energy as well as on A and Z. A linear dependente on A is chosen for simplicity, and the isospin term $4V_1(\mathrm{t}.\mathrm{T})/A$ where t and T are the isospins of the nucleon and the nucleus respectively, gives the required dependence on nuclear symmetry. In the case of protons, there is in addition the usual electrostatic potential together with the term $0.4\ Z/A^{1/3}$ in the expression for V to allow for the effect of the Coulomb field in the presence of an (implicitly) energy-dependent nuclear potential (Perey, *1963).*

The central potential also depends on the isospin of the final state, since proton stripping can go to both $T_>$ and $T_<$ states while neutron stripping can go only to $T_>$ states, and conversely for pickup reaciions.

Assuming pure isospin for all states, the expressions for the potential depths obtained by stripping reactions (particle states) are thus:

$$V_n^> = V_0 - \frac{N-Z}{A} V_1 + \gamma A, \tag{2-2}$$

$$V_p^< = V_0 + \frac{N-Z+2}{A} V_1 + \gamma A + 0.4Z/A^{1/3}, \tag{2-3}$$

$$V_p^> = V_0 + \frac{N-Z}{A} V_1 + \gamma A + 0.4Z/A^{1/3}, \tag{2-4}$$

and the corresponding expressions for pickup reactions (hole states)

$$V_p^> = V_0 + \frac{N-Z+3}{A} V_1 + \gamma A + 0.4(Z-1)/A^{1/3}, \tag{2-5}$$

$$V_n^< = V_0 - \frac{N-Z-1}{A} V_1 + \gamma A, \tag{2-6}$$

$$V_n^> = V_0 + \frac{N-Z+3}{A} V_1 + \gamma A, \tag{2-7}$$

where (N, Z, A) refer to the target nucleus.

The appropriate parametrization for the depth of the spin-orbit potential is more difficult to determine, and the corresponding data is much less precise, since the strength of this potential is determined from the difference between the energies of the $J = L \pm \frac{1}{2}$ states. Where the energies of both these states are known, V and V, can be calculated for a particular form factor, and the resulting values of V, showed no well-marked dependence on any of the available parameters, although they were mostly in the range $5 \leq V_s \leq 10$ MeV. All calculations were therefore made with two assumptions for the spin-orbit potential, namely (A) V, = 7 MeV throughout, (B) V, calculated from the $J = L \pm \frac{1}{2}$ pair if both members available or interpolated from neighbouring nuclei if not.

As the size of the nucleus increases, more and more states move down from positive energies to become bound. The unoccupied (particle) states are reached by stripping reactions, and expressions (2-2) – (2-4) determine the appropriate potential, while the occupied states may be studied by pickup reactions and the potential found from expressions (2-5) – (2-7). The fragmentation of the states introduces the difficulty that some of the single-particle strength may be in particle states and some in hole states. This occurs for the $1f$-state in the present study.

The values of the potential corresponding to the experimental binding energies were obtained by solving the appropriate Schrodinger wave equation and then fitted by the relations (2-2)-(2-7). The results for $2s$ hole states are displayed in Table 1 (Millener and Hodgson, 1971), and for $1d_{\frac{3}{2}}$ hole states in Table 2.

State	V_0	V_1	γ	RMS deviation	N.° of states
Neutron T,	59.5	35.6	–0.209	0.23	13
Proton T,	58.2	38.9	–0.202	0.23	27
Neutron T,	61.9	31.3	–0.218	0.30	9
$N(T_<) + P(T_>)$	58.9	37.2	–0.195	0.23	40

Table 1 - Parameters of bound state potential for 2s hole states (all energies in MeV).

State	V_0	V_1	γ	RMS deviation	N.° of states
Neutron $T_<$	57.4	29.5	–0.169	0.35	14
Proton T,	55.8	41.1	–0.166	0.51	21
Neutron T,	59.9	35.1	–0.203	0.58	10
$N(T_<) + P(T_>)$	57.2	36.1	–0.155	0.47	35

Table 2 - Parameters of bound state potential for $1d_{3/2}$ hole states.

The quality of the fit is indicated in more detail in Fig. 1 which shows the fit to the experimental binding energies as a function of A. The choice of a linear dependence of the potential on A is shown to be adequate by Fig 2. The quality of the fit shows that the energies of the single-particle states vary in a regular and systematic way from nucleus to nucleus.

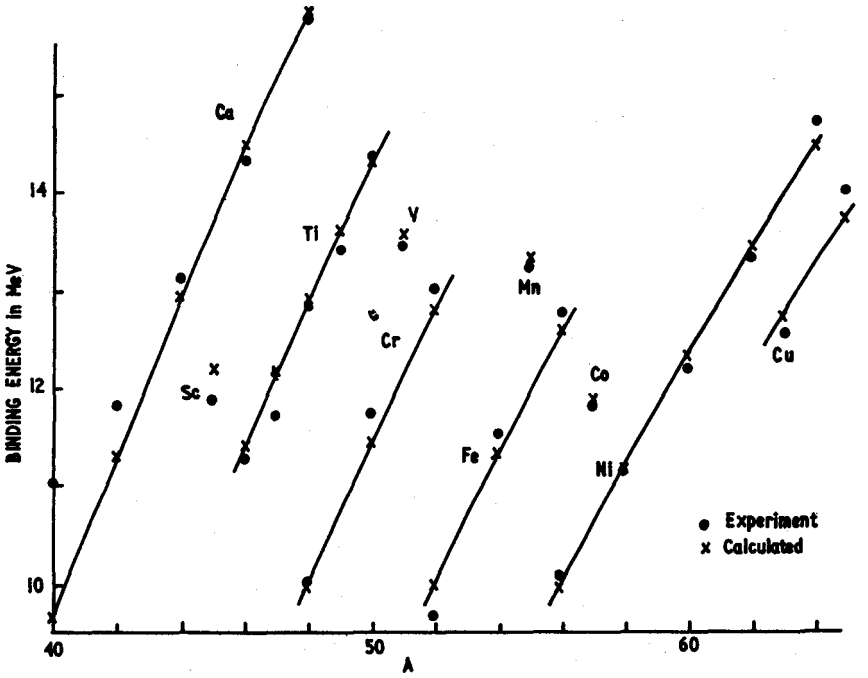

Fig. 1 - Measured energies of $2s_{1/2}$ proton states compared with calculations using the expression (2-5) with the parameters given in the second line of Table 1. The points corresponding to nuclei belonging to the same isotopic sequence are joined by lines. (Millener and Hodgson, 1971).

The results show several notable features. For each value of L, the parameters for all types of states are consistent with each other. The deviations from the average values are consistent with the uncertainties due to the omission of small fragments of the single-particle states.

The value of $V_1 = 37$ MeV obtained for the isospin potential for $2s$ hole states is substantially greater than the generally accepted value of 24 MeV

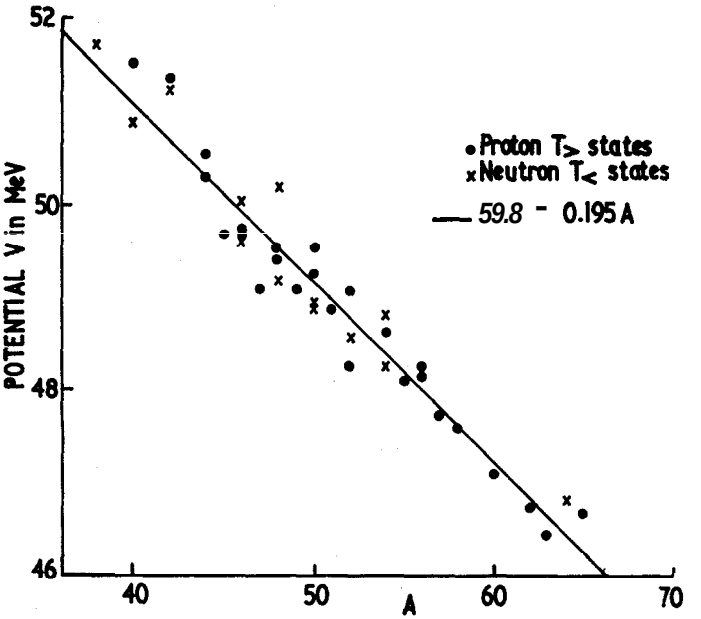

Fig. 2 - The dependence of the nuclear potential V on A after removal of isospin and Coulomb dependences. The parameters have the mean values for neutron $T_<$ and proton $T_>$ states given in the fourth line of Table 1. (Millener and Hodgson, 1971).

for the overall potential (Hodgson, 1964). These values may not be inconsistent, since the isospin potential may show strong state and energy dependence, though at present there is not much evidence for this. It is however interesting to note that the evidence in favour of 24 MeV is now very slight and that recent investigations tend to give a substantially higher value. The main evidence in favour of 24 MeV came from analyses of proton elastic scattering, but it is now known that the dependence of the potential on the symmetry parameter is partly of geometrical and partly of isospin origin, and that these cannot be separated without using additional information on nuclear structure (Hodgson, 1970). (The only exception to this are measurements on isobaric sequences, but no adequate analyses of this type are available).

On the other hand, recent analyses by Kohler (1971) of the single particle energy spectra of a range of nuclei give $V_1 = 32.5$ MeV and Krutov and Savushkin (1969) have obtained $V_1 = 39$ MeV from analyses of heavy deformed nuclei.

It is planned to use the results of these analyses to investigate the state dependence of the bound state nucleon potential, and also to generate nuclear matter and charge distributions for comparison with those obtained in other ways.

3. J-Dependent Sum Rules in Nucleon Transfer Reactions

The most effective way to determine the single-particle characteristics of nuclear states is by nucleon transfer reactions and, by comparing the experimental cross-sections with those calculated by the distorted wave theory, it is possible, in favourable cases, to determine the single-particle strength, or spectroscopic factor, to an absolute accuracy of about 20%. The relative spectroscopic factors of states excited in the same reaction may be found rather more precisely.

It is important to improve the accuracy of these determinations, so that theories of nuclear structure may be tested more rigorously. It is difficult to improve the distorted wave theory without substantial increase in complexity, so it is useful to explore other possibilities, in particular the use of sum rules.

A typical nucleon transfer reaction adds a nucleon with orbital and total angular momenta (l, j) to a nucleus with spin J_T to give a final state of spin J. These angular momenta satisfy the vector relation

$$\mathbf{J} = \mathbf{J}_T + l + \tfrac{1}{2}. \tag{3-1}$$

The case when $J_T = 0$ is particularly simple, for then $J = \mathrm{j} = l \pm \tfrac{1}{2}$. The sum rules relate the spectroscopic factors for stripping and pickup reactions on the same target nucleus; physically these reactions determine the number of holes and the number of particles in a particular state, and their sum is simply $(2\mathrm{j} + 1)$.

This simple picture is somewhat complicated by the need to satisfy the isospin selection rules, and detailed calculations give for neutron transfer reactions the following sum rules:

Neutron Pickup

$$\sum_i S^i_{jl}(T_{F>}) = \frac{1}{N - Z + 1}(\text{protons}) \; ; \tag{3-2}$$

$$\sum_i S^i_{jl}(T_{F<}) = (\text{neutrons}) - \frac{1}{N - Z + 1}(\text{protons}) \; ; \tag{3-3}$$

$$\sum_i S^i_{jl}(T_{F>}) = (\text{neutron holes}), \tag{3-4}$$

$$\sum_i S^i_{jl}(T_{F<}) = 0, \tag{3-5}$$

where the sums run over all the i fragments into which the state is fragmented, and the ( ) indicates the number of particles or holes in the state concerned. Adding the sum rules **(3-2)** to **(3-5)** gives

$$\sum_s S^s_{jl} + \sum_p S^p_{jl} = (\text{neutrons}) + (\text{neutron holes}) = 2j + 1, \tag{3-6}$$

where the indices s and p refer to stripping and pickup reactions respectively.

Similar rules may be obtained for proton transfer reactions. [Thus if measurements of both stripping and pickup reactions are made on the same nucleus the sum rules **(3-6)** may be applied to check the consistency of the analysis]. Departures from the sum rule may be due to fragments being missed, incorrect assignment of quantum numbers, and to uncertainties in the distorted wave theory and in the postulated mechanism of the interaction. It may be difficult to be sure which of these contribute in a particular case, but nevertheless the sum rule does provide an additional constraint that must improve the overall reliability of the analysis.

The sum rules are familiar and have often been used in analyses of nucleon transfer reactions. There are however other sum rules that provide additional constraints for reactions on nuclei with $J_T \neq 0$. Among these are the J-dependent sum rules

$$\frac{2J_s + 1}{2J_T + 1} = S_{Js} + (2J_s + 1)\sum_{JP} (-)^{2j+2J_T} \begin{Bmatrix} J_p & j & J_T \\ J_s & j & J_T \end{Bmatrix} S_{Jp}, \tag{3-7}$$

$$\frac{2J_p + 1}{2J_T + 1} = S_{Jp} + (2J_p + 1)\sum_{Js} (-)^{2j+2J_T} \begin{Bmatrix} J_s & j & J_T \\ J_p & j & J_T \end{Bmatrix} S_{Js}, \tag{3-8}$$

whcre J_s and J_p are the spins of the final states excited in stripping and pickup reactions respectively. It may be noted that addition of **(3-7)** and **(3-8)** gives the previous sum rule **(3-6).** These J-dependent sum rules have been obtained by French **(1966)** and attention has been drawn to their usefulness by Clement **(1971).**

In order to clarify the meaning of these sum rules, it is useful to work out their implications for a few simple cases:

A. $J_T = 1/2,\ l = 0$

In this case j = 1/2 and J_p and J, can take the values 0 and 1, and the sum rules become

$$2S_{s1} + S_{p1} + 3S_{p0} = 3, \tag{3-9}$$
$$2S_{s0} + S_{p1} - S_{p0} = 1, \tag{3-10}$$

where S_{s0} is the sum of the spectroscopic factors for all $l = 0$ stripping transitions to states with $J = 0$ and so on. Thus if S_{p0} and S_{p1} are known, S_{s0} and S_{s1} can be calculated and vice-versa. The sum rules thus provide relations between the spectroscopic factors corresponding to reactions to states of different J.

B. $J_T = 1/2,\ l = 1$

In this case j = 1/2 or 3/2, and the corresponding values of J_p, J_s are (0,1) for j = 1/2 and (1,2) for j = 3/2. The sum rules are:

$j = 1/2$:

$$2S_{s0} - S_{p0} + S_{p1} = 1, \tag{3-11}$$
$$2S_{s1} + 3S_{p0} + S_{p1} = 3; \tag{3-12}$$

$j = 3/2$:

$$2S'_{s1} - S'_{p1} + 3S_{p2} = 6, \tag{3-13}$$
$$4S_{s2} + 5S'_{p1} + S_{p2} = 10. \tag{3-14}$$

C. $J_T = 1,\ l = 1$

In this case j = 1/2 or 3/2, and the corresponding value of J_p, J_s are (1/2, 3/2) for j = 1/2 and (3/2, 5/2) for j = 3/2. The sum rules are:

$j = 1/2$

$$3S_{s1/2} - S_{p1/2} + 2S_{p3/2} = 2, \tag{3-15}$$
$$3S_{s3/2} + 4S_{p1/2} + S_{p3/2} = 4; \tag{3-16}$$

$j = 3/2$

$$6S'_{s1/2} + S'_{p1/2} - 2S'_{p3/2} + 3S_{p5/2} = 4, \tag{3-17}$$
$$15S'_{s3/2} - 10S'_{p1/2} + 11S'_{p3/2} + 6S_{p5/2} = 20, \tag{3-18}$$
$$10S'_{s5/2} + 15S'_{p1/2} + 6S'_{p3/2} + S_{p5/2} = 20. \tag{3-19}$$

Before venturing to use these sum rules as spectroscopic tools, it is important to see how well they are obeyed in situations that are already well understood

These J-dependent sum rules may be tested by comparison with suitable experimental data. Ideally, these should satisfy the following requirements, though sometimes a partia1 check is possible with less complete data:

1. The spin of the target nucleus is non-zero;
2 Data on both nucleon pickup and nucleon stripping are available for the same nucleus;
3. The spectroscopic factors for both the pickup and the stripping reactions are significantly different from zero;
4. The transitions with $j = l \pm \frac{1}{2}$ for the transferred nucleon are distinguished;
5. The J of the final state is known for all transitons;
6. The energy is high enough to excite all the states contributing to the sum rules;
7. It is also desirable that all the final states are bound, to avoid the difficulties connected with the theory of stripping to unbound states.

The determination of the spin of the final state J and of the total angular momentum j of the transferred nucleon presents difficulties. The cross-section of the nucleon transfer reaction is more sensitive to the value of l and this serves only to set limits to J and j. The remaining ambiguities may be resolved by measurement of the polarisation of the outgoing nucleon, or using the J- or j-dependent effects on the differential cross-sections. In some cases it may prove useful to measure the (p, a) or (α, p) reactions to the same final states, as they show a marked J-dependence. It has also been found that the j-dependent effects are stronger in (h, a) than in (p, d) reactions, possibly because the reaction is more concentrated in the surface region (Bohne *et* al., **1970).**

Several cases have been found that enable the J-dependent sum rules to be tested and these are discussed below. They may wnveniently be classified by their values of J_T and l.

1. $J_T = \frac{1}{2}$, $l = 0$

This is the simplest case, but no good examples have yet been found. A likely nucleus is F^{19}, and the study of $F^{19}(d,h)O^{18}$ by Kaschl et *al.* **(1970)** shows three $l = 0$ transfers to $J = 0$ states, giving $S_{po} = 0.58$, and no $l = 0$ transfers to $J = 1$ states, $S_{p1} = 0$. The sum rules **(3-9)** and **(3-10)** give $S_{,} = 0.79$ and $S_{s1} = 0.63$. No complete analysis of the stripping reaction is available, but the work of Ritter *et al.* **(1969)** gives $S_{so} = 1.02$, and three other $l = 0$ transitions to final states of unknown spin have a total S_s of 0.42.

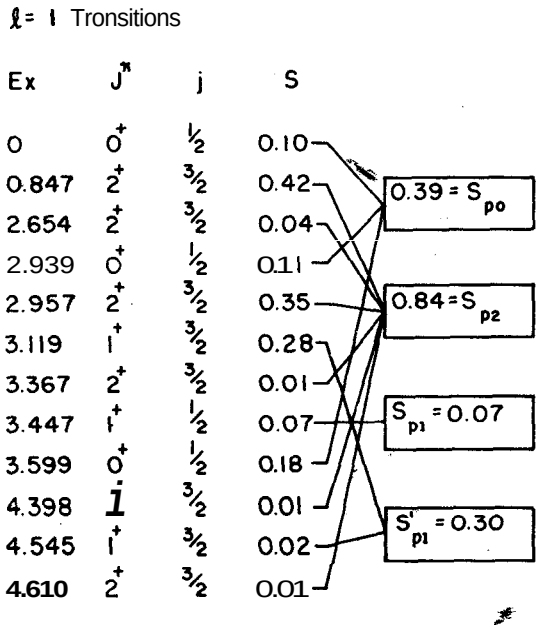

Table 3 - Spectroscopic Factors for the reaction $Fe^{57}(d, t)Fe^{56}$, (Daehnick, *1969).*

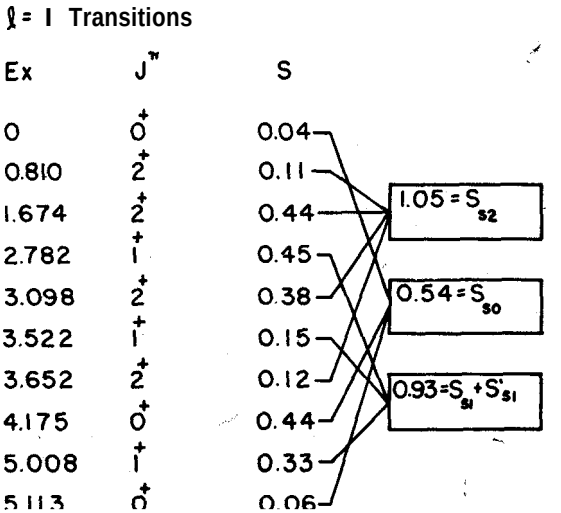

Table 4 - Spectroscopic Factors for the reaction $Fe^{57}(d, p)Fe^{58}$, (Futmer and McCarthy, *1963).*

2. $J_T = \frac{1}{2}$, $l = 1$

This is the next simplest case, and Fe^{57} provides an almost complete example. The results for the pickup and stripping reactions are shown in Tables 3 and 4, and all that is lacking is the assignments for the stripping reactions, so that S_{s1} and S'_{s1} are not separately available, but only their sum. The comparison between the measured spectroscopic factors for stripping and those calculated by the j-dependent sum rules from the spectroscopic factors for the pickup reaction is shown in Table 5. The agreement is encouraging, particularly if the spectroscopic factors are normalised to the overall sum rule total, but the remaining discrepancies merit further investigation.

State	(l, j)	Calculated	Measured	S
0^+	$p_{1/2}$	0.66	0.54(0.94)	S_0
1^+	$p_{1/2}$	0.88 } 1.83	0.93(1.62)	S_1
1^+	$p_{3/2}$	0.92 }		S'_1
2^+	$p_{3/2}$	1.91	1.05(1.84)	S_2

Table 5 - Comparison of Spectroscopic Factors for the reaction $Fe^{57}(d, p)Fe^{58}$ calculated from the data for $Fe^{57}(d, t)Fe^{58}$ (Table 1) and the measured spectroscopic factors (Table 2). The figures in parentheses are obtained by normalising the measured values so as to satisfy the overall sum rule.

3. $J_T = 1$, $l = 1$

The nucleus N^{14} would appear to be suitable for a study of this case, but although there are very many papers devoted to neutron and proton pickup and stripping on this nucleus none is sufficiently detailed for the validity of the sum rules to be investigated.

4. $J_T = \frac{3}{2}$, $l = 0, 2$

A promising nucleus of spin 3/2 is Rb^{87}, and some measurements have been made of the $Rb^{87}(d, p)Rb^{88}$ (Rapaport et al., 1971; Torti and Graetzer, 1971) and $Rb^{87}(d, t)Rb^{86}$ (Dawson et al., 1969) reactions. Several $l = 0$ transitions are observed in the (d, p) reaction but not in the (d, t) reaction, indicating that the S state is completely unoccupied. The $l = 2$ transitions are observed in both reactions, but no spectroscopic factors are available for the (d, *t)* reaction. Application of the sum rules also requires discrimination between the $2d_{3/2}$ and $2d_{5/2}$ neutron transfers.

We thank Dr. C. F. Clement for drawing our attention to the importance of j-dependent sum rules.

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