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The Problem of Bound States in Relativistic Field Theory and its Implications in the Scalar and Pseudoscalar Gluon Model for Quarks*

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The study of bound states in potential theory using Padé approximants in the evaluation of the off-mass shell scattering amplitudes was proposed by Alabiso, Butera and Prosperi. In this case, the convergence of the Padé sequences is substantially enhanced. We propose a similar method for field theoretical models aiming to the development of convenient approximations for the bound states in relativistic quark models.

O estudo de estados ligados na teoria do potencial, utilizando os aproximantes de Padé para as amplitudes de difusão fora da camada de massa, foi proposto por Alabiso, Butera e Prosperi. Neste caso, a convergência das sequências de Padé é bastante aumentada Propomos aqui um método semelhante para modêlos de teoria de campos com a finalidade de desenvolver aproximações convenientes para os estados ligados em modêlos relativísticos de quarks.

1. Introduction

It has been pointed out recently[1] that it is possible to look for bound states in potential theory starting from Padé approximants of scattering amplitudes taken off mass-shell, improving in this way enormously the convergente of the Padé sequence, in comparison with the S matrix (or T matrix) approximants.

The use of a similar method is here proposed and adapted for field theoretical models with the particular purpose of developing convenient approximations for the bound states in relativistic quark models. Indeed, owing to the great binding energy, this seems to be a typical case where the predictions of the S matrix are quite different from those of its lower order Padé approximants.

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In analogy with the work of Alabiso, Butera and Prosperi[1], we must deal with the relativistic four-point function $G(J, s)$ at fixed total J, in order to look for the bound states of the quark-antiquark system as fixed poles in the s variable (s r W^2 c.m.), namely, the square of the total c.m. energy. The complete Green's function for the elastic quark-antiquark system, in perturbation theoiy or in Padé approximation, is a matrix $16 \otimes 16$ in spin space and has also a functional dependence on several variables besides the usual Mandelstam s, t, u. If it has a pole in s, however, this must be common to all matrix elements and we therefore can restrict ourselves to only a part of the whole matrix, namely, the one that corresponds to the general spinor basis $\bar{u}(p') \otimes \bar{v}(q)$, $u(p)$ @ $v(q')$ where p, p', q, q' are here considered the four-momenta of two arbitrary spin 1/2 fermions incoming and outgoing, and two spin 1/2 antifermions respectively, to which we may attribute masses m_1, m_2, m_3, m_4. Such masses appear as free parameters in our Green's function and may vary freely without producing any shift in the s-pole which remains fixed in the exact off-shell amplitudes. This property is the starting point of the variational approach in the Padé approximants.

Our main purpose is then to build the Padé approximants of $G(J, s, m_1, m_2, m_3, m_4)$ from perturbation theory and then use the vanational method mentioned above, on the external masses $m, , m_2, m_3, m_4$ (or on the mass m') in order to derive the bound-state equation $g^2 = g^2(s)$, where g^2 is the renormalized coupling constant of our field model.

To this end, we first remark that renormalizability joined with Lorentz invariance leaves us very little choice for the Langrangian interaction density: we indeed are left with the scalar, pseudoscalar and vector interaction, or explicitly:

$$L_s = g_s \sum_{\alpha=1}^{3} \bar{q}_\alpha(x) q_\alpha(x) \psi(x), \tag{1-1}$$

$$q_\alpha(x) \equiv \text{quark field, } \alpha = 1, 2, 3$$

$$\psi(x) \equiv \text{gluon field (scalar or pseudoscalar),}$$

$$L_{ps} = g_p \sum_{\alpha=1}^{3} \bar{q}_\alpha(x) \gamma^5 q_\alpha(x) \psi(x), \tag{1-2}$$

$$L_v = ig_v \sum_{\alpha=1}^{3} \bar{q}_\alpha(x) \gamma^\mu q_\alpha(x) B_\mu(x), \qquad B_\mu(x) \equiv \text{uector } \textit{gluon} \tag{1-3}$$

(where we take equal-mass quarks for the moment).

Great attention has been devoted to the vector coupling because of its peculiarity of being chiral $SU(3) \otimes SU(3)$ invariant[2]. However, if we restrict ourselves to an $SU(3)$ phenomenological description of the elementary particles, this chiral invariance is already violated by the introduction of the mass terms in the free Lagrangian and so we may also consider the interaction terms that break chirality as the pseudoscalar or the scalar one.

We here formulate the further assumption that the gluon responsible for the forces is an $SU(3)$ singlet and analyze in detail the scalar and the pseudoscalar cases. On the other hand, there seems good evidence in favour of these couplings from the nonrelativistic bosonic and fermionic states describeú by the quark model. More precisely, the fact that the effective mass of the quark appears very small, from quite accurate fits of the mesonic spectrum and from the electromagnetic form factor of the nucleon[3], has an easy explanation with a relativistic scalar potential in the Dirac equation for the quark particle[4] and consequently in terms of scalar or pseudoscalar field couplings. Another point favouring this type of interaction is the fact that it provides attractive forces, with different intensities, for both quark--antiquark and the quark-quark systems, thus providing a basis for the description of the three quarks. This is more difficult to be understood within the frame of the vector coupling.

2. Quantitative Formulation of Quark-Antiquark Bound States

The two body quark-antiquark system is here looked at by studying the off-shell four-point function of the quark-antiquark elastic process. The latter can be calculated as a two body T matrix amplitude $T(J, s)$ with arbitrary spin 1/2 particles initial and final, whereas the "physical" quark appears with its fixed mass m in the propagator lines. The calculation of the $G(J, s)$, is here further simplified by assuming all the external lines with the same mass $m' \equiv m_1 \equiv m_2 \equiv m_3 \equiv m_4 \neq m$ The $G(J, s)$ thus coincides essentially with the partial wave matrix $T(J, s, m')$ which follows from the five helicity amplitudes

$$\phi_1 \equiv T_{+\frac{1}{2}+\frac{1}{2},\,+\frac{1}{2}+\frac{1}{2}}, \qquad \phi_2 \equiv T_{+\frac{1}{2}+\frac{1}{2},\,-\frac{1}{2}-\frac{1}{2}}, \qquad \phi_3 \equiv T_{+\frac{1}{2}-\frac{1}{2},\,+\frac{1}{2}-\frac{1}{2}}$$

$$\phi_4 \equiv T_{+\frac{1}{2}-\frac{1}{2},\,-\frac{1}{2}+\frac{1}{2}}, \qquad \phi_5 \equiv T_{+\frac{1}{2}+\frac{1}{2},\,+\frac{1}{2}-\frac{1}{2}},$$

(for definitions and details see the Appendix).

We have in fact for the singlet and the decoupled triplet transitions (parity $(-1)^{J+1}$):

$$T(J\,|\,0,J;0,J)=4\pi F_1(J,s), \tag{2-1}$$

$$T(J\,|\,1,J;1,J)=4\pi\{F_4(J,s)+(2J+1)^{-1}[JF_3(J+1,s)+ \\ +(J+1)\cdot F_3(J-1,s)]\}, \tag{2-2}$$

and again for the coupled triplet T matrix elements (parity $(-1)^J$):

$$T(J\,|\,1,J-1;1,J-1)=(2J+1)^{-1}[(J+1)\,h^J_{1,1}+J\,h^J_{2,2}- \\ -2\sqrt{J(J+1)}\,h^J_{1,2}]\equiv T_{11}, \tag{2-3}$$

$$T(J\,|\,1,J+1;1,J-1)=(2J+1)^{-1}[\sqrt{J(J+1)}\,(h^J_{1,2}-h^J_{1,1})- \\ -h^J_{1,2}\,J]\equiv T_{12}\equiv T_{21}, \tag{2-4}$$

$$T(J\,|\,1,J+1;1,J+1)=(2J+1)^{-1}[(J+1)\,h^J_{1,1}+J\,h^J_{2,2}- \\ -2\sqrt{J(J+1)}\,h^J_{1,2}]\equiv T_{22}, \tag{2-5}$$

where

$$h^J_{1,2}=4\pi\,F_2(J,s), \tag{2-6}$$

$$h^J_{2,2}=4\pi\{F_3(J,s)+(2J+1)^{-1}[J\,F_4(J+1,s)+(J+1)\,F_4(J-1,s)]\}, \tag{2-7}$$

$$h^J_{1,2}=4\pi[\sqrt{J(J+1)}/(2J+1)]\,[F_5(J+1,s)-F_5(J-1,s)], \tag{2-8}$$

and the partial wave functions $F_k(J,s)$ are defined as $(1/2)\int_{-1}^{+1} dzP_J(z)F_k(s,t)$;

the F_k are linearly related to the ϕ's in the following fashion:

$$F_1=\phi_1-\phi_2\equiv\hat{\phi}_1-\hat{\phi}_2, \tag{2-9}$$

$$F_2=\phi_1+\phi_2\equiv\hat{\phi}_1-\hat{\phi}_2, \tag{2-10}$$

$$F_3=(1/2)(\hat{\phi}_3-\hat{\phi}_4), \tag{2-11}$$

$$F_4=(1/2)(\hat{\phi}_3+\hat{\phi}_4), \tag{2-12}$$

$$F_5=\hat{\phi}_5, \tag{2-13}$$

with

$$\phi_k=\hat{\phi}_k[\sin\,(\sigma/2)]^{|a-b|}\cdot[\cos(\sigma/2)]^{|a+b|}. \tag{2-14}$$

(see Appendix for definitions of a, a, *b)*

These formulas represent the kinematical framework which relates the partial waves to the invariant amplitudes for equal mass spin 1/2 two-body scattering and must be used in perturbation theory to evaluate the Padé approximants of $T(J, s, m')$.

The simplest approximant $T^{(1,1)}$ consists of the Born term (T_B) and the full fourth-order renormalized calculation (T_4); one must then compute the helicity amplitudes that are explicitly given in the Appendix for both the scalar and the pseudoscalar interaction, at the required perturbative order.

3. Padé Approximants

Dealing with two spin 1/2 particles in the initial and final states, the T matrix, as it has been pointed out in the kinematical formulas, has two different representations, in the partial waves, for the two opposite parity states. More specifically, once the $SU(3)$ quantum numbers are given, we have the singlet ($J = l$, $S = 0$) and one of the triplets ($J = l$, $S = 1$) as simple one-dimensional amplitudes with the total parity equal to $(-1)^{J+1}$, which differ from each other for the charge conjugation quantum number $C = (-1)^{S+l}$, in the non-strange quark-antiquark bound states and are decoupled. These amplitudes, however, become coupled through the singlet-triplet transition in the strange quark-antiquark states when the mass difference between the λ and the n-p quarks is taken into account. The other possibility is the two by two transition matrices that correspond to the coupled triplet (from $J \pm 1 \rightarrow J \pm 1$) with parity $(-1)^J$, as it is easily seen from formulas (2-3), (2-4), (2-5). To such a structure for the T matrix elements must be joined the wanted $SU(3)$ physical states in order to provide us with the various bosonic states we are looking for.

Let us consider the Padé approximants in the uncoupled ($J = l$) and in the coupled cases. The former gives a simple expression for the $T^{(1,1)}(J, s, m')$.

$$T^{[1,1]}(J, s, m') = \frac{g^2 T_B(J, s, m')}{1 - \dfrac{g^2 T_4(J, s, m')}{T_B(J, s, m')}}. \tag{3-1}$$

The latter brings to the two by two matrix

$$T^{[1,1]}(J, s, m') = g^2 T_B \cdot [T_B - g^2 T_4]^{-1} \cdot T_B. \tag{3-2}$$

The bound state function $g^2(s, m')$ is therefore equal to T_B/T_4 in the uncoupled case and to

$$(-\Delta_{B4} \pm \sqrt{(\Delta_{B4})^2 - \Delta_4\Delta_B}) \cdot \frac{1}{\Delta_4}$$

for the coupled triplet, where

$$\Delta_4 \; r \; \det \begin{vmatrix} (T_4)_{11} & (T_4)_{12} \\ (T_4)_{12} & (T_4)_{22} \end{vmatrix}, \tag{3-3}$$

$$\Delta_B \; E \; \det \begin{vmatrix} (T_B)_{11} & (T_B)_{12} \\ (T_B)_{12} & (T_B)_{22} \end{vmatrix}, \tag{3-4}$$

and

$$4 \quad = -(T_4)_{11}\,(T_B)_{22} - (T_4)_{22}\,(T_B)_{11} + 2\,(T_4)_{12}\,(T_B)_{12} \tag{3-5}$$

The variational method toward the parameter m' is now performed on the function $g^2(s, m')$ in order to obtain the stationary function $\bar{g}^2(s)$ such that

$$\delta_{m'}\; \bar{g}^2(s, m') \equiv 0. \tag{3-6}$$

Once the function $\bar{g}^2(s)$ is obtained, the bosonic spectrum and the corresponding Regge trajectories must be looked at.

In practice the stationary point can be searched only numerically by tabulating g as function of m'.

4. Bosonic Spectrum and Regge Trajectories

The formulas explicity treated here are valid for the bosonic octet states and, with the introduction of the symmetry-breaking effects in the coupling or in the masses, they may be used for a large fit of particles; the singlet states can also be considered by adding the corresponding graphs with only gluons in the intermediate states.

The evaluation of the Regge trajectories is simple from formulas (2-1) – (2-5), because of the possibility of considering for the functions $F_k(J, s)$ the Froissart-Gribov representation from our formulas that are written by dispersive integrals. All formulas of ours do reggeize except for the $J = 0$ case, where we may find Kronocker delta functions in our partial wave *T*-matrix elements; correspondingly we have the blowing up of the Froissart-Gribov representation. The investigation of the Regge trajectories is very interesting in the equal mass case (m, $= m_{n,p}$) because several features of these are supposed to be independent of the $SU(3)$ symmetry-breaking effects.

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Notations and Definitions

μ = gluon mass
m = quark mass
m' = external line mass
$t = -2p^2(1-\cos a)$
$p^2 = (1/4)(s - 4m'^2)$
$\sin^2 \frac{\sigma}{2} = -\frac{t}{4p^2}$
$\cos^2 \frac{\sigma}{2} = 1 + \frac{t}{4p^2}$

$k' = -p/\sqrt{s}$
$c = i\pi^2 = i\,3\,!\,\pi^2 B(2,2)$
$c' = (1/2)\,i\pi^2 = i\frac{3\,!}{4}\pi^2 B(3,1)$
$s = -(p+q)^2 = -(p'+q')^2$
$t = -(p'-p)^2$
$u = -(p'-q)^2$

Appendix

The Scalar Interaction

Born term — The Bom term (second order in g) is essentially a gluon exchange in the *t*-channel, because the gluonic exchange in the s-channel contributes only to the $SU(3)\,q\bar{q}$ singlet states. We obtain indeed:

$$\phi_1 = k'\cos^2 \frac{\rho}{2} : \frac{-m'^2}{\mu^2 - t} : \left(\frac{g}{2\pi}\right)^2,$$

$$\phi_2 = k' \sin^2 \frac{\sigma}{2} \cdot \frac{s}{4} \cdot \frac{1}{\mu^2 - t}\left(\frac{g}{2\pi}\right)^2,$$

$$\phi_3 = +\phi_1, \qquad \phi_4 = -\phi_2,$$

$$\phi_5 = k' \sin\sigma \cdot m' \frac{\sqrt{s}}{4}\,\frac{1}{\mu^2 - t}\left(\frac{g}{2\pi}\right)^2$$

Fourth order — It consists of the self-energy part, the two vertex (equal) parts and the direct and crossed box diagrams. Again, we treat only the contributions that are relevant for the octets.

Self **energy — It equals three times (n,p,λ quark loops) the following contribution:**

$$\phi_1 = k' \cos^2 \frac{\sigma}{2} \cdot (-m'^2)\, \eta(t) \,.\left\{ i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

$$\phi_2 = k' \sin^2 \frac{\sigma}{2} \cdot \frac{s}{4} \cdot \eta(t) \cdot \left\{ i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

$$\phi_3 = \phi_1 , \qquad \phi_4 = -\phi_2 ,$$

$$\phi_5 = k' \sin \sigma \cdot m' \cdot \frac{\sqrt{s}}{4} \eta(t) \cdot \left\{ i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

where

$$\eta(t) = \frac{1}{2\pi i} \int_{4m^2}^{+\infty} dt' \frac{\Delta\eta(t')}{t'-t}$$

$$\Delta\eta(t') = -(2\pi)^{-3} \frac{4m^2 - t'}{(\mu^2 - t')} \frac{\sqrt{t' - 4m^2}}{t'}$$

Vertex part — It is twice the following contribution:

$$\phi_1 = k' \cos^2 \frac{\sigma}{2} (-m'^2)\, \nu(t) \cdot \left\{ -i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

$$\phi_2 = k' \sin^2 \frac{\sigma}{2} \cdot \frac{s}{4} \cdot \nu(t) \cdot \left\{ -i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

$$\phi_3 = \phi_1 , \qquad \phi_4 = -\phi_2 ,$$

$$\phi_5 = k' \sin \sigma \cdot m' \cdot \frac{\sqrt{s}}{4} \nu(t) \cdot \left\{ -i(2\pi)^{-2} \left(\frac{g}{2\pi}\right)^4 \right\},$$

where

$$\nu(t) = \frac{1}{2\pi i} \int_{4m^2}^{+\infty} dt' \frac{\Delta\nu(t')}{t'-t},$$

$$\Delta\nu(t) = \quad -t)^{-1} \left\{ (m + m')^2 + \mu^2 - 2m'(m + m') \frac{m^2 - m'^2}{t - 4m'^2} + 4\mu^2 m'(m + m')_{\overline{t - 4m'^2}} \right\} \cdot \Delta\psi(t)$$

$$+ (\mu^2 - t)^{-1} \left\{ -1 - 4m' \frac{(m + m')}{t - 4m'^2} \right\} \cdot \Delta\tilde{\psi}(t),$$

$$\Delta\psi(t) = +\pi \cdot (2\pi)^2 \frac{1}{\sqrt{t}\sqrt{t - 4m^2}} Q_0 \left(\frac{t/2 - m'^2 - m^2 + \mu^2}{(1/2)\sqrt{t - 4m'^2}\sqrt{t - 4m^2}} \right),$$

$$\Delta\tilde{\psi}(t) = -(2\pi)^2 \cdot \pi \cdot \frac{\sqrt{t - 4m^2}}{t}.$$

Direct box diagram

$$\phi_1 = c\phi_1^A + c'\phi_1^B, \quad \phi_1^A = k'\left(1 + \frac{t}{s-4m'^2}\right) g_1 \cdot \left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_1 = -m'^2 (m + m')^2 \Psi_A + (m + m')\left(2m'^3 - m'\frac{s}{2}\right)\Psi_B - \frac{1}{2} m'^2 (2m'^2 - s)\Psi_C - \frac{1}{4} s^2 \Psi_D,$$

$$\phi_1^B = k'\left[\frac{1}{2}(2m'^2 - s) - m'^2 \frac{t}{s-4m'^2}\right]\Psi \cdot \left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

$$\phi_2 = c\phi_2^A + c'\phi_2^B, \quad \phi_2^A = k'\left(\frac{-t}{s-4m'^2}\right) g_2 \cdot \left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_2 = \frac{s}{4}(m + m')^2 \Psi_A - m'^2 \cdot \frac{s}{4}\Psi_C + m'^2 s \Psi_D,$$

$$\phi_2^B = k'\frac{-m^2 t}{s-4m'^2}\Psi \cdot \left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

$$\phi_3 = c\phi_1^A + c' k'\left[-\frac{1}{2}(2m'^2 - s)\right]\left[1 + \frac{t}{s-4m'^2}\right]\Psi\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

$$\phi_4 = -\phi_2, \quad \phi_5 = c\phi_5^A + c'\phi_5^B;$$

$$\phi_5^A = k' \sin o \frac{1}{s-4m'^2} \cdot g_5 \cdot \left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_5 = m'(m + m')^2 \frac{\sqrt{s}}{4}\Psi_A + \frac{\quad}{16}(m + m')(s - 4m'^2)\Psi_B - m'\frac{s\sqrt{s}}{16}\Psi_C + \frac{\sqrt{s}}{4} s m' \Psi_D,$$

where

$$\Psi = \int_0^1 dx_1 \int_0^{x_1} dx_2 \int_0^{x_2} dx_3 \, 1/\Delta,$$

$$\Delta = t(x_3^2 - x_2 x_3) + m'^2 x_2 (x_2 - 1) + m^2 (1 - x_2) - s(x_1 - x_2)(1 - x_1) + \mu^2 x_2$$

and dispersively:

$$\Psi = \frac{1}{\pi}\int_{4\mu^2}^{+\infty} dt' \, Im_t \Psi(t', s)/(t' - t)$$

$$Im_t \Psi = -4\pi \cdot \frac{t + 2(m^2 - m'^2 - \mu^2)}{(t + s - 4m'^2)\sqrt{t}\sqrt{t-4m'^2}} \cdot Q_0\left(\frac{t + 2(m^2 - m'^2 - \mu^2)}{\sqrt{t-4m'^2}\sqrt{t-4\mu^2}}\right)$$

$$+ 4\pi\frac{\sqrt{t-4\mu^2}}{t}\frac{1}{t + s - 4m'^2}\sqrt{1 - \frac{sm}{s}} Q_0\left(\sqrt{1 - \frac{s_M}{s}}\right)$$

where

$$s_M = 4\frac{(m'^2 - m^2 + \mu^2)^2 + m^2 t - 4\mu^2 m'^2}{t - 4\mu^2},$$

$$\Psi_A = -\frac{\partial}{\partial m^2}\Psi - \frac{\partial}{\partial \mu^2}\Psi, \quad \Psi_B = -\frac{\partial}{\partial m^2}\Psi,$$

$$\Psi_C = -\frac{\partial}{\partial m^2}\Psi - \frac{\partial}{\partial m'^2}\Psi, \quad \Psi_D = \frac{\partial \Psi}{\partial s}.$$

Crossed box diagram. We obtain:

$$\phi_1 = c\phi_1^A + c'\phi_1^B, \quad \phi_1^A = k'\cos^2\frac{\sigma}{2}\cdot g_1\cdot\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_1 = -m'^2(m+m')^2\Psi_A^C + \frac{sm'}{2}(m+m')\Psi_B^C + \frac{1}{2}m'^2(2m'^2 - s)\Psi_C^C - \left(\frac{s}{2} - 2m'^2\right)^2\Psi_D^C$$

$$\phi_1^B = -k'\left[\frac{1}{2}(2m'^2 - s) - m'^2\frac{t}{s - 4m'^2}\right]\Psi^C\cdot\left\{-i(2)^{-2}\left(\frac{g}{2}\right)^4\right\}.$$

$$\phi_2 = c\phi_2^A + c'\phi_2^B, \quad \phi_2^A = k'\sin^2\frac{\sigma}{2}\cdot g_2\cdot\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_2 = (m+m')^2\frac{s}{4}\Psi_A^C - 2m'(m+m')\Psi_B^C\cdot\frac{s}{4} + m'^2\frac{s}{4}\Psi_C^C,$$

$$\phi_2^B = -k'\sin^2\frac{\sigma}{2}\Psi^C\cdot\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

$$\phi_3 = c\phi_1^A + c'k'\left[\frac{1}{2}(2m'^2 - s)\right]\left[1 + \frac{t}{s - 4m'^2}\right]\Psi^c.$$

$$\phi_4 = -\phi_2$$

$$\phi_5 = c\phi_5^A + c'\phi_5^B$$

$$\phi_5^A = k'\sin\sigma\frac{1}{s - 4m'^2}g_5\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$g_5 = \frac{\sqrt{s}}{4}m'(m+m')^2\Psi_A^C - \frac{\sqrt{s}}{16}(m+m')(s+4m'^2)\Psi_B^C + \frac{s\sqrt{s}m'}{16}\Psi_C^C,$$

$$\phi_5^B = k'\sin\sigma + \frac{1}{2}m'\frac{\sqrt{s}}{2}\cdot\Psi^C\cdot\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

Pseudoscalar Interaction

Born Term

$$\phi_1 = 0, \quad \phi_2 = k' \sin\frac{\sigma}{2}\cdot\frac{-p^2}{\mu^2 - t}\cdot\left(\frac{g}{2\pi}\right)^2,$$

$$\phi_3 = o, \quad 44 = 42, \quad 45 = 0, \quad \phi_6 = 0.$$

Self energy

$$\phi_1 = 0, \quad \phi_2 = k' \sin\frac{\sigma}{2}\cdot(+p^2)\cdot\eta(t)\cdot\left\{i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\},$$

$$\phi_3 = 0, \quad \phi_4 = \phi_2, \quad \phi_5 = 0,$$

where the **function** $\eta(t)$ **was defined in** the **self-energy** section of the scalar case.

Vertex (ps). We obtain

$$\phi_1 = \phi_3 = \phi_5 = o,$$
$$\phi_2 = \phi_4 = k' \sin^2\frac{\sigma}{2}(4m'^2 - s)\frac{1}{4}\mu(t)\cdot\left\{-i(2\pi)^{-2}\left(\frac{g}{2\pi}\right)^4\right\}.$$

$$\mu(t) = (2\pi i)^{-1}\int_{4m^2}^{+\infty} dt' \frac{\Delta\mu(t')}{t' - t},$$

$$\Delta\mu(t) = (\mu^2 - t)^{-1}\{[(m' - m)^2 + p^2]\,\Delta\psi(t) - \Delta\tilde{\psi}(t)\},$$

where $\Delta\psi$ and $\Delta\tilde{\psi}$ are already **defined** in the scalar **interaction** case.

Boxes (ps): Their formulas coincide with the corresponding **scalar** ones with the substitution $m \to -m$.

Ψ_h^c derives from Ψ_n with the substitution $s \to u$ *(h: A, B, C, D)*

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