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The Hartree-Fock and the Random Phase Approximations Applied to Ne^{20}, Si^{28} and Ar^{36}: Energy Levels and E2 Transitions

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The Hartree-Fock approximation was applied to Ne^{20}, Si^{28} and Ar^{36} using different types of two-body interactions. Wave functions with good J were determined with the Peierls-Yoccoz method and they were used to calculate energy levels and electric quadrupole transition probabilities within the ground state band. The intrinsic excited states were calculated with the Random Phase Approximation. From comparison with the available experimental data, one finds that the best results occur for Ne^{Z0} and when Kuo's renormalized effective two-body interaction is used.

A aproximação de Hartree-Fock foi aplicada aos núcleos de Ne^{20}, Si^{Z8} e Ar^{36} usando diferentes tipos de interações de dois corpos. Foram determinadas funções de onda com bom número quântico J pelo método de Peierls e Yoccoz; as mesmas foram utilizadas para calcular os níveis de energia e as probabilidades de transição de quadrupolo elétrico entre níveis da banda fundamental. *Os* estados excitados intrínsecos foram calculados com a "Random Phase Approximation". Da comparação com os dados experimentais disponíveis, verifica-se que os melhores resultados ocorrem para o Ne^{20} e quando se usa a interação efetiva renormalizada de Kuo.

1. Introduction

In recent years, the Hartree-Fock (HF) approximation has been providing a very powerful tool in the study of nuclear structure properties.

The Hartree-Fock wave function is not an eigenstate of the total angular momentum operator and therefore one has to project from it wave functions with good J. Once these wave functions are known, one can calculate transition matrix elements and expectation values of some relevant operators. Most of this formalism can ben found in two reference papers by G. Ripka[1].

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One knows that the formula

$$E_J = E_0 + AJ(J+1) \tag{1-1}$$

works very well for some strongly deformed nuclei.

The existence of a rotational band suggests that the nucleus Hamiltonian H could be divided into two parts, an intrinsic one H_i, plus a rotational term AJ^2, such that

$$H_i = H - AJ^2. \tag{1-2}$$

If a wave function $|\phi^J\rangle$ is an eigenstate of H corresponding to the above eigenvalue E,, it can be easily seen that a linear combination

$$|\phi\rangle = \sum_J C_J |\phi^J\rangle \tag{1-3}$$

will be an eigenstate of the intrinsic Hamiltonian H_i, but not of H, and the states $|\phi^J\rangle$ will be a degenerate set of eigenstates of H_i.

These states $|\phi^J\rangle$ can be projected from the intrinsic state $|\phi\rangle$ and their degeneracy could be removed by adding to H_i the rotational term AJ^2. The Hartree-Fock theory has been used to obtain an approximation to the intrinsic state $|\phi\rangle$.

We have calculated the energy levels of the lowest band as

$$E_J \equiv \langle\phi^J|H_i + AJ^2|\phi^J\rangle = \langle\phi^J|H_i|\phi^J\rangle + AJ(J+1), \tag{1-4}$$

where. the states $|\phi^J\rangle$ are projected from the Hartree-Fock wave function $|\phi\rangle$ which gives the lowest expectation value of **(1-2).**

It becomes very important to know a value for the inertial parameter **A.** In Section *3,* we summarize the derivations of some formulas for this parameter using the cranking model.

Thouless and Valatin[2] (TV) were the first ones to point out the importance of including field effects in the moment of inertia calculation. Slight modifications of their formula were introduced by Banerjee, de Oliveira and Stephenson[3] **(BDS).** Recently, J. **M.** Cohenca and **S. K.** M. Wong *et al.*[4] have independently suggested that another kind of self-consistency should be considered as will be shown in Section **3.** Low-lying excited states will be formed by the superposition of configurations in which one particle is promoted to a previously non occupied level, leaving a hole. These par-

ticle-hole excited states can be treated with the Tamm-Dancoff (TD) or the Random Phase Approximation[5] (RPA), which are built on the static HF solution. The RPA equations can also be derived from the formalism of the time dependent Hartree-Fock approximation[6].

We have applied the Hartree-Fock theory and some subsequent approximations to the nuclei of Ne^{20}, Si^{28} and Ar^{36}. In this calculation we consider the nucleus of O^{16} as an inert core, and the states available to the extra-core particles are confined to the s-d shell, only. Our aim is to obtain: (a) the lowest band spectrum of energy by the projection method; (b) the electric quadrupole transition probabilities between these energy levels; (c) the same lowest band spectrum using the rotational formula $E_J = \mathrm{E}, + + AJ(J + 1)$, with the appropriate value of the moment of inertia $\mathscr{I} = 1/2A$; (d) the low-lying one particle-one hole excited states belonging to different bands. These results are obtained with some different types of effective interactions and compared with the available experimental data We are applying these calculations to nuclei in the s-d shell which are not strongly deformed. Their low-lying energy levels do not constitute a pure rotational spectrum. However, they still exhibit a good degree of deformation and formula (1-1) might be approximately valid Besides, the difficulties in the calculations can be better handled in the case of these low medium mass nuclei than for the heavy ones.

2. Summary of the Hartree-Fock Theory and the Projection Method

The determination of the Hartree-Fock single-particle states, through the solution of the HF equations, are well presented elsewhere[1].

Our Hartree-Fock wave function is expressed in a second quantization notation as

$$|\phi\rangle = a^{\dagger}_{i_1} a^{\dagger}_{i,} \ldots . a^{\dagger}_{i_N} |0\rangle, \tag{2-1}$$

where the states $|i_1\rangle$, $|\mathrm{i},)$ $\ldots . |\mathrm{i},)$ are the HF single-particle occupied orbitals; $|0\rangle$ is the vaccuum state with respect to the fermion operators $a^{\dagger}_{\eta}$ and a,. The number of particles available is N; if $\eta \leq$ N, it refers to an occupied orbital being denoted with the letters i, j; if $\eta >$ N, it refers to an unoccupied orbital m, n.

We use the Hamiltonian operator in the standard form

$$H = \sum_{pq} \langle p\,|t|\,q\rangle\, a^{\dagger}_{p} a_{q} + \tfrac{1}{4} \sum_{pqrs} \langle pr\,|v|\,qs\rangle\, a^{\dagger}_{p} a^{\dagger}_{r} a_{s} a_{q}, \tag{2-2}$$

where the indices p, q,..., refer to any set of single-particle states, in particular to the HF s.p. states \?). The two body interaction matrix elements are antisymmetrized.

In order to assure that the expectation value of H becomes a minimum, it is necessary to impose the condition that

$$\langle 1p-1h\,|H|\,\phi\rangle = 0, \tag{2-3}$$

known as the Hartree-Fock condition.

One can calculate the expectation value of H in $|\,\phi\rangle$, obtaining

$$E \equiv \frac{\langle\phi\,|H|\,\phi\rangle}{\langle\phi\,|\,\phi\rangle} = \sum_{i=1}^{N}\langle i\,|t|\,i\rangle + \frac{1}{2}\sum_{i_1,i_2=1}^{N}\langle i_1\,i_2\,|\,v\,|\,i_1\,i_2\rangle, \tag{2-4}$$

where the relation $a_i^{\dagger}|\phi\rangle = 0$ was used.

G_{JT}	Tri-Axial $\langle H\rangle$	Tri-Axial $\langle Q_0^2\rangle$	Axial Prolate $\langle H\rangle$	Axial Prolate $\langle Q_0^2\rangle$	Axial Oblate $\langle H\rangle$	Axial Oblate $\langle Q_0^2\rangle$
			Ne^{20}			
K	–37.79	–7.72	–37.79	15.44	–29.19	–8.00
K_b	–34.23	–7.90	–34.23	15.60	–30.12	–8.00
KLS	–38.28	–7.86	–38.28	15.70	–34.36	–8.00
S	–34.54	–7.82	–34.54	15.62	–31.20	–8.00
D	–32.30	–7.80	–32.30	15.60	–26.98	–8.00
Y	–37.01	–7.83	–37.01	15.66	–28.66	–8.00
			Si^{28}			
K	–143.84	–11.82	–143.84	23.64	–147.57	–22.96
K_b	–141.47	–11.66	–141.47	23.32	–144.17	–23.04
KLS	–147.63	–11.56	–147.63	23.12	–150.77	–23.06
S	–134.61	–11.20	–134.61	22.40	–138.05	–22.64
D	–137.52	–11.30	–137.52	22.60	–140.36	–22.74
Y	–119.62	–10.70	–119.62	20.60	–120.63	–22.72
			Ar^{36}			
K	–281.91	7.18	–278.09	7.36	–281.91	–14.34
K_b	–296.33	7.30	–292.12	7.46	–296.33	–14.58
KLS	–289.21	6.90	–286.15	7.34	–289.21	–13.90
S	–266.38	6.49	–264.74	7.18	–266.38	–12.94
D	–295.10	6.80	–292.60	7.42	–295.10	–13.92
Y	–209.29	6.20	–207.36	6.88	–209.29	–12.42

Table I - The energies and expectation values of the mass quadrupole operator for the Hartree-Fock ground states with different symmetries. The units are MeV and the oscillator length parameter, respectively. They were calculated with various sets of G_{JT} reduced m. e. and the labels refer to the two-body interactions described in Section 5.

The expression *(2-4)* gives the Hartree-Fock energy of a system of N particles when the states $|i\rangle$ are chosen to give the lowest possible value of E.

The eigenfunctions $|\phi^J_{MK}\rangle$ of the total angular momentum are projected from the Hartree-Fock wave function *(2-1)* using the Peierls-Yaccoz method[7]:

$$|\phi^J_{MK}\rangle = \frac{2J+1}{8\pi^2}\,\frac{1}{C_{JK}}\int d\Omega\, D^{J*}_{MK}(R)R(\Omega)|\phi_K\rangle, \tag{2-5}$$

where R stands for the Euler angles a, β, y,

$$R(\Omega) = e^{-i\alpha J_z}e^{-i\beta J_y}e^{-i\gamma J_z}, \tag{2-6}$$

and

$$D^J_{MK}(R) = \delta_{JJ'}\langle JM\,|R(\Omega)|\,J'K\rangle. \tag{2-7}$$

Indices M and K are the projection of the total angular momentum on the z-axis of the laboratory system and body system, respectively. The nuclei we are investigating have axially symmetric HF solutions $|\phi_K\rangle$, and C_{JK} is a normalization constant.

It can be verified[1] that an irreducible tensor operator will have the following matrix elements between these projected states:

$$\langle\phi^{J_1}_{M_1K_1}|T^{(k)}_q|\phi^{J_2}_{M_2K_2}\rangle = \frac{2J_2+1}{2C^*_{J_1K_1}C_{J_2K_2}}\langle J_2\,M_2\,kq\,|J_1\,M_1\rangle\sum_{\mu\nu}\langle J_2\,K_2\,k\nu\,|J_1\,K_1\rangle$$

$$\int_0^\pi d\beta\sin\beta\, d^{J_1}_{K_1\mu}(\beta)\langle\phi_{K_1}|e^{-i\beta J_y}T^{(k)}_\nu|\phi_{K_2}\rangle, \tag{2-8}$$

where

$$|C_{JK}|^2 = \frac{2J+1}{2}\int_0^\pi d\beta\sin\beta\, d^J_{KK}(\beta)\langle\phi_K|e^{-i\beta J_y}|\phi_K\rangle, \tag{2-9}$$

and

$$d^J_{KM}(\beta) = \langle JK|e^{-i\beta J_y}|JM\rangle = \text{real}. \tag{2-10}$$

Energy of the State $|\phi^J_{MK}\rangle$

This energy is defined as the expectation value

$$E_{JK} = \langle \phi^J_{MK} | H | \phi^J_{MK} \rangle$$
$$= \frac{2J+1}{2|C_{JK}|^2} \int_0^\pi d\beta \sin\beta \, d^J_{KK}(\beta) \langle \phi_K | e^{-i\beta J_y} H | \phi_K \rangle, \qquad (2\text{-}11)$$

which is obtained from the expression (2-8) with k = q = 0, and is independent of M.

As we are interested in the lowest K = 0 band we have

$$E_J = \frac{\int_0^\pi d\beta \sin\beta \, d^J_{00}(\beta) \langle \phi_0 | e^{-i\beta J_y} H | \phi_0 \rangle}{\int_0^\pi d\beta \sin\beta \, d^J_{00}(\beta) \langle \phi_0 | e^{-i\beta J_y} | \phi_0 \rangle} \qquad (2\text{-}12)$$

$$= E_{HF} + \frac{\int_0^\pi d\beta \sin\beta \, d^J_{00}(\beta) \sum_{2p-2h} \langle \phi_0 | e^{-i\beta J_y} | 2p-2h \rangle \langle 2p-2h | H | \phi_0 \rangle}{\int_0^\pi d\beta \sin\beta \, d^J_{00}(\beta) \langle \phi_0 | e^{-i\beta J_y} | \phi_0 \rangle}$$

The Hartree-Fock condition eliminates the one particle-one hole contributions from the above expansion.

B(E2) Transitions within the K = 0 Band

The electric transition probability corresponding to the emission of a photon of energy $\hbar\omega$, angular momentum L, the nucleus going from an initial state $|i\rangle$ to a final state $|f\rangle$, is

$$T_{if}(L) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{e^2 k^{2L+1}}{A} B(EL; i \to f), \qquad (2\text{-}14)$$

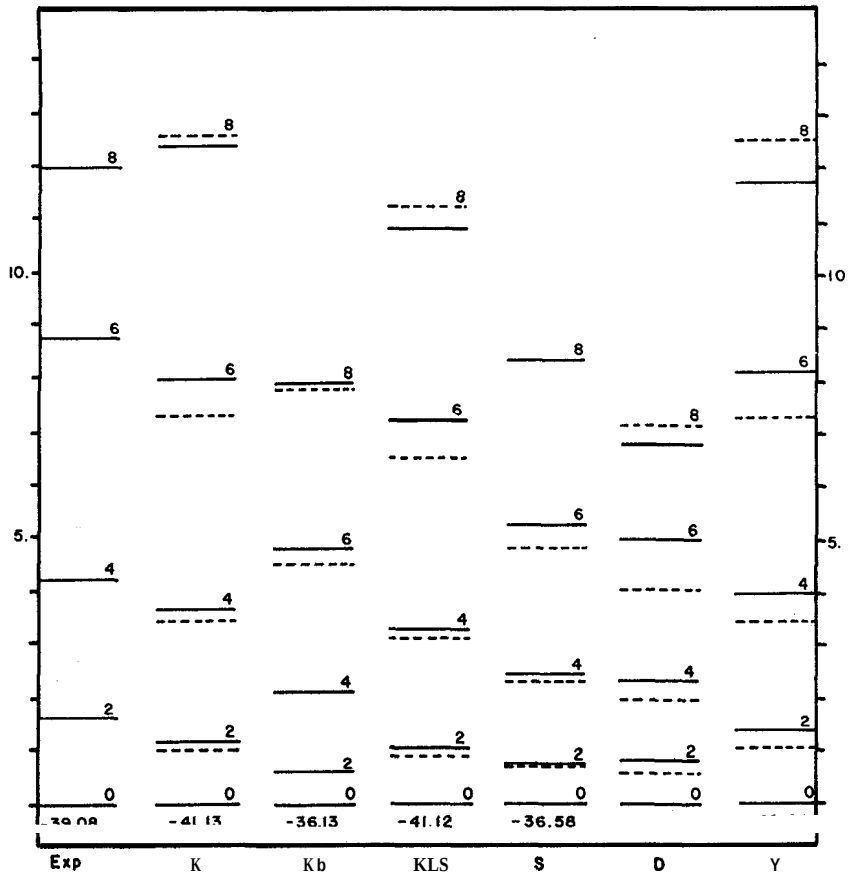

Figure 1 - Projected HF spectra (full lines) of Ne^{20} using different types of two-body interactions. The spin value is on the right of each level. The number in the bottom is the energy of the lowest $J^{\pi} = 0^{+}$ level relative to the O^{16} binding energy. The dashed levels were obtained with the formula $E_J = E_J + A^{(n)} J(J+1)$. In both cases we used the iterated $A^{(n)}$ values of Table II. The energies are in MeV.

where $k = \omega/c$ and B is the reduced matrix elements containing the operator which characterizes the transition from $|i\rangle = |\phi^{J_2}_{M_2 K=0}\rangle$ to $|f\rangle = |\phi^{J_1}_{M_1 K=0}\rangle$, which is

$$B(E2;\ J_2 \to J_1) = \frac{1}{2J_2+1} \sum_{M_1 M_2 q} |\langle \phi^{J_1}_{M_1} | r^2\, Y_{2q} | \phi^{J_2}_{M_2} \rangle|^2 \qquad (2\text{-}15)$$

$$= \frac{(2J_2+1)(2J_1+1)}{4 |C_{J_1}|^2 |C_{J_2}|^2} [\mathcal{G}(J_1, J_2)]^2. \qquad (2\text{-}16)$$

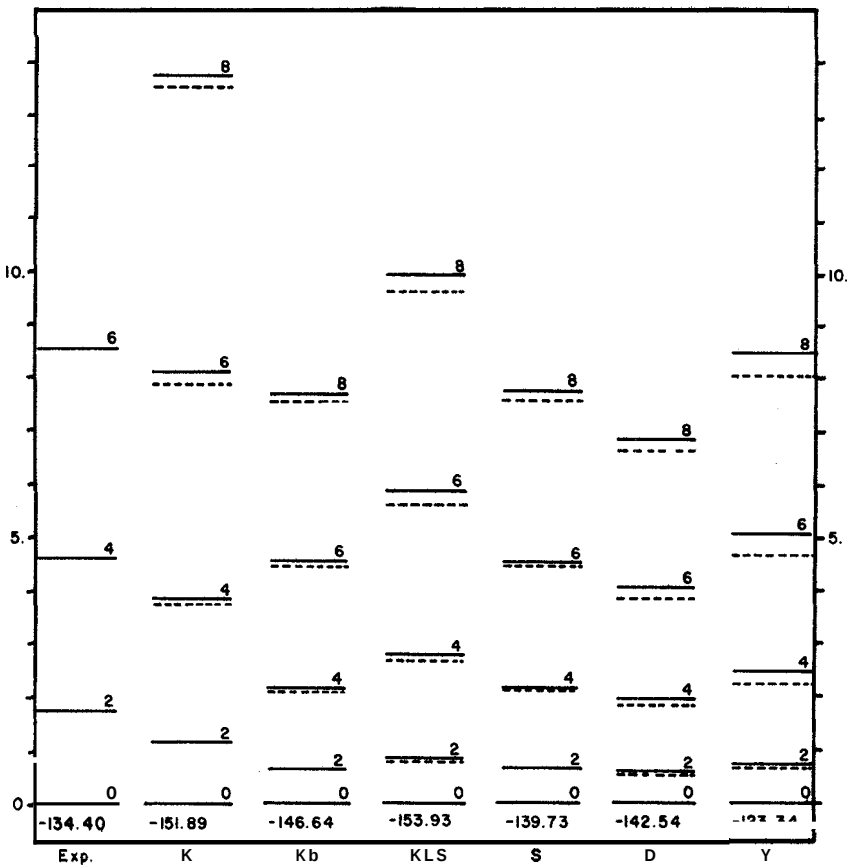

Figure 2 - Hartree-Fock spectra of Si^{28} calculated with different types of two-body interactions. The energies are in MeV. See Figure 1 caption.

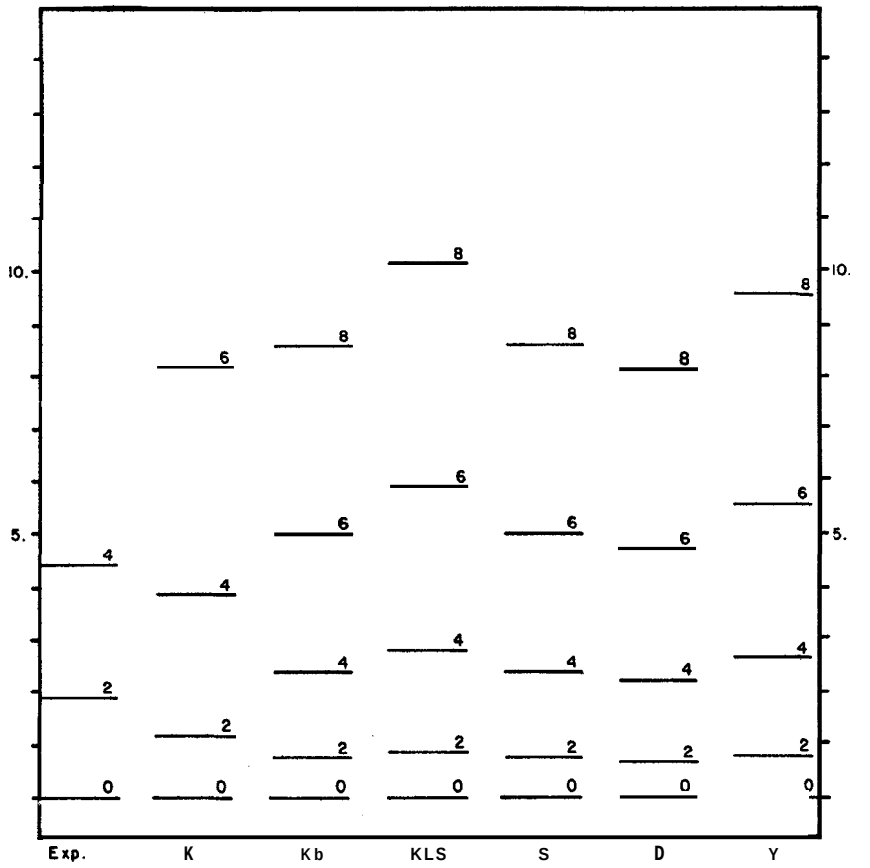

Figure 3 - Energy levels of Ar^{36} calculated with the formula $E_J = E_0 + A^{(n)} J(J+1)$. The iterated $A^{(n)}$ values are from Table II for different types of two-body interactions. The energies are in MeV.

In the above equations, the expression (2-8) was again used with $K_1 = K_2 = 0$, k = 2 along with the orthonormalization properties of the Clebsch-Gordan coefficients. Since $r^2 Y_{2q}$ is a one-body operator, it will be seen that

$$\mathcal{G}(J_1, J_2) = \sum_{\nu=-2}^{2} (J_2\, O\, 2\nu \,|\, J_1\, \nu) \int_0^{\pi} d\beta \sin\beta\, d^{J_1}_{o\nu}(\beta) \times$$
$$\times \{\langle\phi_0|e^{-i\beta J_y}|\phi_0\rangle\langle\phi_0|r^2\, Y_{2\nu}|\phi_0\rangle +$$
$$\sum_{1p-1h} \langle\phi_0|e^{-i\beta J_y}|1p-1h\rangle\langle 1p-1h|r^2\, Y_{2\nu}|\phi_0\rangle\}. \quad (2\text{-}17)$$

3. Formulas for the Moment of Inertia

Consider the nucleus Hamiltonian H perturbed by a term $-\omega J_x$ where J_x is the x- component of the total angular momentum and o is a small paramater. If $|\phi\rangle$ and $|\phi_\omega\rangle$ denote the HF determinantal wave functions corresponding to the ground state of H and

$$H, = H - \omega J_x, \quad (3\text{-}1)$$

respectively, they can be related by the Thouless Theorem[2] as

$$|\phi_\omega\rangle = e^{\omega S}|\phi\rangle, \quad (3\text{-}2)$$

where S is a sum of 1 particle-1 hole terms, only, and does not depend on o. The new Hamiltonian H, can be interpreted as representing a deformed axially symmetric nucleus subject to a small rotation about an axis perpendicular to the symmetry axis. Banerjee *et al.*[3] obtained the Thouless--Valatin equation for the operator S as follows: first, they write the expectation value of H, in $|\phi_\omega\rangle$ as a power series in o using relations (3-1) and (3-2), getting

$$\langle\phi_\omega|H_\omega|\phi_\omega\rangle = \langle\phi|H|\phi\rangle + \frac{\omega^2}{2}\langle\phi|[S,[S,H]] + 2[S,J_x]|\phi\rangle + \cdots, \quad (3\text{-}3)$$

since one can verify that the linear term vanishes.

To assure that the l.h.s. of equation (3-3) is a HF minimum, the coefficient of o^2 is minimized with respect to variations of S, resulting the equation

$$2\langle\phi|[\delta S, \{[S,H] + J_x\}]|\phi\rangle +$$
$$+ \langle\phi|[\delta S,[H,S]] + [S,[\delta S,H]] + [H,[S,\delta S]]|\phi\rangle = 0. \quad (3\text{-}4)$$

G_{JT} / $J_i^\pi \to J_f^\pi$	EXP.	K		K_b		KLS		S		D		Y	
		PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.
$2^+ \to 0^+$	17.8 ± 2.5	17.7	(17.7)	18.1	(18.1)	15.5	(15.5)	15.4	(15.4)	20.3	(20.3)	15.5	(15.5)
$4^+ \to 2^+$	21.9 ± 2.1	22.5	25.3	22.9	25.8	19.7	22.1	19.5	22.0	25.7	29.0	19.6	22.1
$6^+ \to 4^+$	20.6 ± 2.4	19.1	27.9	19.4	28.5	16.7	24.4	16.5	24.2	21.8	31.9	16.6	24.3
$8^+ \to 6^+$		11.4	29.2	11.5	29.8	9.8	25.5	9.7	25.4	12.8	33.4	9.7	25.4

Table II – $B(E2)$ strengths in Weisskopf units (W.U.) for the ground state band of Ne^{20}, For different types of two-body interactions. The expe nmental data are from Ref. 22. The PHF column contains the values calculated with projected HF wave functions. In the other column the results from the adiabatic rotationai model are shown. The number in parenthesis is normalized to the PHF value.

G_{JT} / $J_i^\pi \to J_f^\pi$	EXP.	K		K_b		KLS		S		D		Y	
		PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.	PHF	Adiab.
$2^+ \to 0^+$	14 ± 4	22.7	(22.7)	22.9	(22.9)	24.6	(24.6)	23.7	(23.7)	24.1	(24.1)	23.9	(23.9)
$4^+ \to 2^+$	21 ± 5	30.7	32.4	30.8	32.8	33.2	35.2	32.0	33.9	32.7	38.3	32.2	34.1
$6^+ \to 4^+$		30.2	35.7	30.4	36.0	32.6	38.8	31.5	37.2	32.0	37.8	31.7	37.6
$8^+ \to 6^+$		26.1	37.4	26.3	37.8	28.2	40.6	27.3	39.0	27.7	39.6	27.4	39.4
$10^+ \to 8^+$		18.9	38.4	19.1	38.8	20.5	41.7	19.7	40.1	20.1	40.7	19.8	40.4
$12^+ \to 10^+$		10.4	39.1	10.5	39.4	11.3	42.4	10.9	40.8	11.3	41.4	11.0	41.2
$14^+ \to 12+$		*23.3*	*39.6*	*22.5*	*39.9*	*24.8*	*43.0*	*24.6*	*41.3*	*24.9*	*41.6*	*24.6*	*41.7*

Table III – $B(E2)$ strengths in W.U. for the ground state band of Si^{28}. (See caption of Table II). The experimental data are from Ref. 23.

The term $\langle\phi|[\delta S,[S, H]]|\phi\rangle$ was simply added and subtracted; also the term $\langle\phi|[H, [S,\delta S]]|\phi\rangle$ was added, the latter being equal to zero due to the Hartree-Fock condition (2-3), because the commutator [S, δS] is an $1p-1h$ operator. As the positions of the three operators obey a circular permutation, the sum of the commutators in the second bracket of (3-4) vanishes. We are then left with

$$\langle\phi|[\delta S,\{[S,H]+J_x\}]|\phi\rangle. \tag{3-5}$$

Since δS is an arbitrary 1p – lh operator, one gets the Thouless-Valatin equation for the operator S:

$$\langle 1p-1h|[H,S]|\phi\rangle = \langle 1p-1h|J_x|\phi\rangle. \tag{3-6}$$

The only non-vanishing matrix elements of S are those between the occupied states $|i)$ and unoccupieú states $|m)$. Using S as

$$S=\sum_{mi}\{\mathscr{S}_{mi}\,a_m^\dagger\,a_i-\mathscr{S}_{mi}^*\,a_i^\dagger\,a_m\} \tag{3-7}$$

and the form (2-2) for the Hamiltonian, we get the following two equations corresponding to the explicit form of Eq. (3-6):

$$\begin{aligned}
&\sum_{nj}\{(\varepsilon_m-\varepsilon_i)\,\delta_{ij}\,\delta_{mn}+\langle mj|v|in\rangle\}\,\mathscr{S}_{nj}+\sum_{nj}\langle mn|v|ij\rangle\,\mathscr{S}_{nj}^*=\langle m|J_x|i\rangle,\\
&\sum_{nj}\{(\varepsilon_m-\varepsilon_i)\,\delta_{ij}\,\delta_{mn}+\langle mj|v|in\rangle^*\}\,\mathscr{S}_{nj}^*+\sum_{nj}\langle mn|v|ij\rangle^*\,\mathscr{S}_{nj}=\langle i|J_x|m\rangle,
\end{aligned} \tag{3-8}$$

where the two-body interaction matrix elements are antisymmetrized.

Using the relation (3-2), the HF condition (2-3) and (3-6) we may write the change of the energy of the nucleus due to the perturbation as

$$\begin{aligned}
\Delta E\equiv\langle\phi_\omega|H|\phi_\omega\rangle-\langle\phi|H|\phi\rangle &\cong \tfrac{1}{2}\omega^2\langle\phi|[S,[S,H]]|\phi\rangle\\
&=\tfrac{1}{2}\omega^2\,\langle\phi|[J_x,S]|\phi\rangle.
\end{aligned} \tag{3-9}$$

By equating ΔE to $\frac{1}{2}\mathscr{I}\omega^2$, we obtain the Thouless-Valatin formula for the inertial parameter $A=\dfrac{1}{2\mathscr{I}}$, which is

$$A_{TV}=\frac{1}{2\langle\phi|[J_x,\,S]|\phi\rangle}. \tag{3-10}$$

On the other hand, the inertial parameter is defined in Ref. 3 as being essentially the ratio of AE to the change of the expectation value of J^2, due to the perturbation,

$$A_{BDS} = \frac{\langle \phi \,|\, [J_x,\ S] \,|\, \phi \rangle}{\langle \phi \,|\, [S, [S, J^2]] \,|\, \phi \rangle}. \tag{3-11}$$

The term $\langle \phi \,|[S,\ J^2]\,|\, \phi \rangle$ does not contribute to $\Delta\langle J^2 \rangle$ because the HF wave function $|\, \phi \rangle$ is assumed to be invariant under time reversal and it can be shown that the operator S is odd under this symmetry operation.

J. M. Cohenca[4] proposed that one should proceed iteratively to get the inertial parameter through formulas (3-10) or (3-11). This same idea was proposed by Ng, Trainor and Wong[4] which have used a formula due to Skyrme for the inertial parameter. Their suggestion is based on the argument that the Hamiltonian $H_i = \mathrm{H} - \mathrm{AJ}^2$ would have a better chance to represent the nucleus intrinsic state. The procedure is the following: Once some formula is chosen, calculate the first value $A^{(1)}$ with the HF wave function $|\, \phi_i^{(0)} \rangle$ obtained with the initial Hamiltonian $H_i^{(0)} = H - A^{(0)} J^2$; in particular, $H_i^{(0)}$ could be the original H, $(A^{(0)} = 0)$. Next, construct the new Hamiltonian $H_i^{(1)} = \mathrm{H} - A^{(1)}\mathrm{J}^2$ which will generate the HF wave function $|\, \phi_i^{(1)} \rangle$; using the same inertial parameter formula previously chosen, calculate $A^{(2)}$ with $H_i^{(1)}$ and $|\, \phi_i^{(1)} \rangle$. Repeat the process until some value $A^{(n)}$ becomes equal to $A^{(n-1)}$, the one obtained previously.

To get the Inglis formula in this context, neglect the two-body matrix elements in (3-8) and substitute the expression for $\mathscr{S}_{mi}$ into the formula (3-10), resulting

$$\mathscr{I}_{Ing} \equiv \frac{1}{2\, A_{Ing}} = 2 \sum_{mi} \frac{\langle m \,|\, J_x \,|\, i \rangle^2}{\varepsilon_m - \varepsilon_i}. \tag{3-12}$$

The important contribution to the moment of inertia theory was brought by Thouless and Valatin, who have shown that the operator S should be the solution of the complete equation (3-8). Once we use this correct solution for S, the formulas (3-10), (3-11) or the iterative calculation proposed in the papers of Ref. 4, are equally acceptable. We feel that the self-consistency required in the iterative procedure should give a better result.

G_{JT}	$A^{(o)} = 0$			$A^{(n)} = A^{(n-1)}$		
	BDS	TV	ING.	BDS	TV	ING.
			Ne^{20}			
K	0.183	0.149	0.243	0.175	0.175	0.232
K_b	0.114	0.093	0.188	0.109	0.109	0.179
KLS	0.168	0.137	0.234	0.157	0.157	0.218
S	0.123	0.102	0.191	0.116	0.116	0.178
D	0.108	0.089	0.163	0.100	0.100	0.150
Y	0.192	0.159	0.314	0.173	0.173	0.277
			Si^{28}			
K	0.195	0.175	0.251	0.187	0.187	0.233
K_b	0.109	0.097	0.179	0.105	0.105	0.171
KLS	0.139	0.125	0.190	0.134	0.134	0.179
S	0.117	0.106	0.163	0.113	0.113	0.154
D	0.097	0.087	0.152	0.092	0.092	0.145
Y	0.113	0.103	0.191	0.111	0.111	0.183
			Ar^{36}			
K	0.203	0.162	0.241	0.193	0.193	0.247
K_b	0.123	0.098	0.185	0.118	0.118	0.190
KLS	0.144	0.114	0.170	0.140	0.140	0.179
S	0.123	0.098	0.137	0.118	0.118	0.147
D	0.118	0.094	0.152	0.112	0.112	0.159
Y	0.143	0.116	0.192	0.131	0.131	0.195

Table IV - Values, in MeV, of the inertial parameters for Ne^{20}, Si^{28} and Ar^{36} calculated with the formulas presented in Section 3. The G_{JT} labels refer to the interactions described in Section 5.

4. Low-lying Excited States. The RPA Equations

For an extensive and detailed discussion of this formalism, see Ref. 5. We give here a brief summary containing only some of the formulas which entered in the calculation.

Consider an operator $B_\lambda(t)$ satisfying the Heisenberg equation

$$i\hbar \frac{\partial B_\lambda(t)}{\partial t} = [B_\lambda(t), H]. \tag{4-1}$$

Assuming its time dependence to be of the usual form

$$B_\lambda(t) = B_\lambda e^{-i\omega_\lambda t}, \tag{4-2}$$

equation (4-1) becomes

$$[\mathrm{B}, , \mathrm{H}] = \mathrm{ho}, \mathrm{B}, . \tag{4-3}$$

The ground-state wave function and energy should satisfy the equation

$$H\,|\psi_{G.S.}\rangle = E_{G.S}\,|\psi_{G.S}\rangle. \tag{4-4}$$

From (4-3) and (4-4), it can be verified that

$$H\,B_\lambda\,|\psi_{G.S.}\rangle = (E_{G.S.} - \hbar\omega_\lambda)\,B_\lambda\,|\psi_{G.S.}\rangle, \tag{4-5}$$

and

$$H\,B_\lambda^\dagger\,|\psi_{G.S.}\rangle = (E_{G.S.} + \hbar\omega_\lambda)\,B_\lambda^\dagger\,|\psi_{G.S.}\rangle. \tag{4-6}$$

Assuming that the excitation energy $\hbar\omega_\lambda$ is real and positive, the definition of the ground state must be completed with the condition that

$$B_\lambda\,|\psi_{G.S.}\rangle = 0, \qquad \text{for all } \lambda, (\mathrm{ho}, > 0). \tag{4-7}$$

The case $\hbar\omega_\lambda < 0$ can be treateú analogously starting with the equation for $B_\lambda^\dagger$.

If B, is a one-body operator, by taking the matrix elements between the HF state $|4)$ and $|\phi_\alpha\rangle = a_m^\dagger a_i\,|\phi\rangle$ of both sides of equation (4-3), one obtains

$$\begin{aligned} \sum_\beta \{\mathscr{A}_{\alpha\beta}\,X_\beta^\lambda + \mathscr{B}_{\alpha\beta}\,Y_\beta^\lambda\} &= \hbar\omega_\lambda\,X_\alpha^\lambda, \\ \sum_\beta \{\mathscr{B}_{\alpha\beta}^*\,X_\beta^\lambda + \mathscr{A}_{\alpha\beta}^*\,Y_\beta^\lambda\} &= -\hbar\omega_\lambda\,Y_\alpha^\lambda, \end{aligned} \tag{4-8}$$

which can be more conveniently written as

$$\begin{pmatrix} \mathscr{A} & \mathscr{B} \\ \mathscr{B}^* & \mathscr{A}^* \end{pmatrix} \begin{pmatrix} X^\lambda \\ Y^\lambda \end{pmatrix} = \hbar\omega_\lambda \begin{pmatrix} X^\lambda \\ -Y^\lambda \end{pmatrix}. \tag{4-9}$$

These are the RPA equations, where

$$\begin{aligned} X_\alpha^\lambda &= \langle\phi\,|B_\lambda|\,\phi_\alpha\rangle, \\ Y_\alpha^\lambda &= -\langle\phi_\alpha\,|B_\lambda|\,\phi\rangle, \end{aligned} \tag{4-10}$$

$$\mathscr{A}_{\alpha\beta} = (E, \; -\varepsilon_i)\,\delta_{ij}\,\delta_{mn} + (\text{in } |v|\,mj), \tag{4-11}$$

$$\mathscr{B}_{\alpha\beta} = \langle ij\,|v|\,mn\rangle, \tag{4-12}$$

T	Band number	RPA			TD	Exp.
		Y	K	KLS	KLS	
0	0	4.16	6.67	5.72	5.78	6.72
0	0	10.56	10.02	9.12	9.19	
0	0	12.57	12.15	11.56	11.60	
0	0	13.79	12.69	13.16	13.21	
0	1	0	0	0	2.26	
0	1	10.30	9.79	9.07	9.10	9.95
0	1	12.62	12.11	11.73	11.78	
0	1	15.41	15.62	15.17	15.19	
0	2	6.52	7.63	7.39	7.40	7.43
0	2	9.68	9.09	8.92	8.92	
0	2	14.25	14.09	13.47	13.49	
0	3	10.73	11.12	10.26	10.30	
1	0	10.55	10.65	9.61	9.61	
1	0	11.10	11.22	10.42	10.55	
1	0	13.92	13.07	12.37	12.37	
1	0	14.10	13.52	12.90	12.99	
1	1	10.03	9.14	8.65	8.68	
1	1	10.76	10.88	9.92	9.96	
1	1	14.21	13.58	12.92	12.96	
1	1	15.61	15.85	14.87	14.90	
1	2	9.71	8.57	7.87	7.90	10.27
1	2	11.09	11.31	10.35	10.36	
1	2	15.85	16.32	15.14	15.15	
1	3	10.74	10.88	9.80	9.80	

Table V – RPA excitation **energies** $\hbar\omega_\lambda$, in **MeV**, for Ne^{20} with the interactions (Y), (K) and (KLS). The Tamm-Danwff approximation (6^{th} wlumn) **gives practically** the **same** results except for the spunous state. The experimental data are from ***Nuclear Data Sheets*** (1961) except for the **level** at **9.95 MeV [Ref. 22].** The **first column contains** the isotopic spin states.

and ε_m, ε_i correspond to the HF single particle unoccupied and **occupied** states, respectively. The **quantities** $\mathscr{A}_{\alpha\beta}$ and $\mathscr{B}_{\alpha\beta}$ have already **appeared** in the explicit **form** of the **Thouless-Valatin** equations (3-8).

The RPA ground-state **energy** is **given** by

$$E_{RPA} = E_{HF} - \tfrac{1}{2}\Big(\sum_{\alpha} \mathscr{A}_{\alpha\alpha} - \sum_{\lambda>0} \hbar\omega_\lambda\Big), \tag{4-13}$$

Band Number	$T=O$ K	Exp.	$T=1$ K	Exp.
0	9.02	4.97	10.45	
0	9.60		11.01	
0	11.28		12.19	
0	11.76		12.32	
0	13.82		16.31	
0	15.41		17.06	
1	0		10.96	
1	7.71	7.38	11.71	
1	11.26		12.32	
1	12.14		14.65	
1	14.83		15.80	
1	16.19		16.61	
2	9.63	7.41	11.06	9.38
2	11.06		11.16	
2	13.03		14.21	
2	14.22		15.78	
2	18.01		19.16	
3	5.89	6.27	11.14	9.32
3	12.77		14.39	
3	18.24		18.94	
4	13.53	6.89	13.65	

Table VI – RPA excitation energies $\hbar\omega_\lambda$, in MeV, for Si^{28} using Kuo's (K) interaction The experimental data are from Ref. 27.

Band number	$T=0$ K	Exp.	$T=1$ K	Exp.
0	6.21	4.33	9.34	
0	7.69		9.43	
0	10.95		13.24	
0	11.83		13.61	
1	0		7.84	
1	8.10	5.83	9.47	
1	11.75		13.00	
1	12.54		14.05	
2	5.28	4.95	7.97	6.61
	11.41		13.41	
	14.27		15.49	
	14.69	7.18	15.07	

Table VII - RPA energies in MeV for Ar^{36}. See Table VI caption.

so that the correlations introduced by the RPA into the Hartree-Fock ground-state can be measured by the energy difference

$$E_{RPA} - E_{HF} = -\tfrac{1}{2}(\sum_{\alpha} \mathscr{A}_{\alpha\alpha} - \sum_{\lambda>0} \hbar\omega_{\lambda}), \tag{4-14}$$

which can be proved to be negative.

B. Jancovici and D. H. Schiff[8] obtained this same expression using the Hill – Wheeler – Griffin Method of Generator Coordinates.

	Ne^{20}		Si^{28}		Ar^{36}	
G_{JT}	E_{RPA}	$E_{CORR.}$	E_{RPA}	$E_{CORR.}$	E_{RPA}	$E_{CORR.}$
K	–40.58	–2.79	– 152.24	–4.67	– 285.00	– 3.09
K_b	– 35.90	– 1.67	– 147.00	–2.83	–298.08	– 1.75
KLS	–40.76	– 2.48	– 154.53	–3.76	–291.54	–2.33
S	– 36.32	– 1.78	– 141.11	– 3.06	–268.33	– 1.95
D	–34.04	–1.74	–143.43	–3.07	–297.13	– 2.03
Y	– 39.68	–2.67	–124.78	–4.15	–211.13	– 2.04

Table VIII - RPA ground state energies and the correlation energies for Ne^{20}, Si^{28} and Ar^{36} calculated with different interactions. All values are in MeV.

5. The Interactions

In order to perform the calculation, one must know the fwo-body matrix elements of the nucleon-nucleon interaction in the nucleus. Usually they are written in the basis of the harmonic oscillator single-particle wave functions, If the interaction is rotational invariant and charge independent, they can be expressed as

$$\begin{aligned}\langle j_1 m_1 \tau_1\, j_2 m_2 \tau_2 \,|v|\, j_3 m_3 \tau_3\, j_4 m_4 \tau_4\rangle = \sum_{\substack{JM \\ TT_z}} & (j_1 m_1\, j_2 m_2 \,|\, JM) \times \\ & \times (j_3 m_3\, j_4 m_4 \,|\, JM)\,(\tfrac{1}{2}\tau_1\, \tfrac{1}{2}\tau_2 \,|\, TT_z)\,(\tfrac{1}{2}\tau_3\, \tfrac{1}{2}\tau_4 \,|\, TT_z) \\ & \times [(1+\delta_{j_1 j_2})(1+\delta_{j_3 j_4})]^{1/2}\, G_{JT}(j_1 j_2\,,\, j_3 j_4).\end{aligned}$$

In this paper, we use six types of interactions that enter in the calculation as different sets of the quantity G_{JT}, which are known as the antisymmetrized and normalized particle-particle reduced matrix elements of the interaction. These G_{JT} can be taken as the effective interactions determine from realistic two-body forces that fit the nucleon-nucleon scattering

data The techniques involved in the determination of the effective interaction are the ones developed by Brueckner and others in 'the theory of nuclear matter. The first quantitative calculations of these matrix elements were performed by Kuo and Brown[9], and Kuo[10] using the Hamada-Johnston potential Their results were restricted to the s-d shell and this truncation of the shell-model space implied that renormalization corrections should be taken into account. Kuo's effective-interaction matrix elements are the sum of G_{bare} plus these corrections; we used them in this calculation and the results appear under the label (K).

Recently, Kirson and Zamick[11] have shown that when some of these renormalization effects are consistently calculated in all orders, their final contribution becomes very small compared to G_{bare}. Therefore, we also performed the calculation using Kuo's bare reaction matrix (K_b).

Two other sets of bare effective-interaction matrix elements were used: One was obtained by S. Kahana, H. C. Lee and C. K. Scott[12] (KLS) from a non-local but separable potential, similar to the one of Tabakin.

Recently, J. Demos[13] solved Brueckner's G-matrix equation starting from Reid's soft-core two-body potential. We also used this effective interaction as G_{JT} matrix elements and the results have the label (*D*).

Next, it was possible to use as G_{JT} the Sussex (S) matrix elements derived by Elliott et *al.*[14]. They were obtained directly from the scattering phase shifts without specifying an explicit form for the two-body potential. The Sussex matrix elements were transformed into the j-j coupling representation by C. Abulafio[15] and we have used them in this latter form.

Finally, a much simpler interaction was used, namely a Yukawa (Y) central force with a range of 1.4 fm. The parameters of the exchange mixture were adjusted[16] to fit the low-lying level spectra of O^{18} and F^{18}. The strengths in the various space spin states are: triplet-even, –43.0 MeV; singlet-even, – 31.5 MeV; triplet-odd, 17.8 MeV; singlet odd, 37.4 MeV.

Of all sets of matrix elements, only the one of Kuo (*K*) includes renormalization corrections.

6. Results and Discussions

In Table I, we show the Hartree-Fock energies corresponding to three choices of symmetry solutions; each symmetry was built in at the beginning

of the iterative process and carried through each step of the iteration. The same energy value may occur for two symmetry choices; when this happened we have always worked with the axially symmetric solution having the lowest energy. Therefore, Ne^{20} is prolate, Si^{28} and Ar^{36} are oblate with axial symmetry, as it has been known since the work of Bar-Touv and Kelson[17]. We also display the expectation values of the mass quadrupole operator $Q_{20} = \sqrt{\frac{16\pi}{5}}\, r^2\, Y_{20}$ in the HF ground-state, expressed in units of the oscillator-length parameter.

The single-particle shell model energies $\varepsilon_{ij} = \varepsilon_i \delta_{ij}$ were taken from the experimental spectra of O^{15} and O^{17} neglecting Coulomb energy differences. All energies are measured in MeV relative to the O^{16} binding energy, which is supposed to behave as an inert core. The matrix elements of the two-body interaction were described in Section 5. With (K) and (K_b) we designate Kuo's effective interaction, the first containing renormalization corrections while the second is the bare G-matrix. Next, we have Kahana, Lee and Scott (*KLS*) separable potential, then the Sussex (S), Demos (D) and Yukawa (Y) two-body matrix elements; none of these last four include renormalization effects.

Oscillator radial wave functions that have a length parameter $b = \sqrt{\frac{\hbar}{m\omega}}$ were used. The value of ω should change according to the mass number. However, Kuo's (K) and Demos's (D) matrix elements were available to us only for fixed values of fio, 14.0 MeV (b = 1.72 fm) and 13.25 MeV (b = 1.77 fm), respectively. In each case the same set of G_{JT} was used for Ne^{20}, Si^{28} and Ar^{36}. The other four types of reduced matrix elements could be obtained for any desired value of $\hbar\omega$ which was fixed acwrding to the formula $\hbar\omega = 41\ A^{-1/3}$ MeV, giving 15.1 MeV, 13.5 MeV and 12.4 MeV for each of those nuclei in that sarne order.

The projection calculation was performed with the Peierls-Yoccoz method, for Ne^{20} and Si^{28} and the results for the spectrum of energy are shown in Fig. 1 and Fig. 2. We worked with the intrinsic Hamiltonian $H_i = H - A^{(n)} J^2$ obtaining the energy expectation values

$$E_J \equiv \langle \phi^J | H | \phi^J \rangle = \langle \phi^J | H_i | \phi^J \rangle + A^{(n)} J(J+1), \qquad (6\text{-}1)$$

where $A^{(n)}$ is the inertial parameter reached after few iterations (n = 3, 4) with either TV or BDS formulas. One can find in the literature a great number of papers showing this kind of result for a variety of forms of

the two-body interaction[18]. While for Ne^{20} the results can be considered good, it has been known that the Hartree-Fock approximation for Si^{2}' provides an intrinsic wave function which seems to be too deformed; in this case, the calculated spectrum of energy tends to be largely compressed compared to the experimental one. Not all interactions we used are able to provide a good comparison with experiment, even for Ne^{20}. The best fit is obtained with Kuo's (K) G-matrix including renormalization corrections; the (KLS) interaction gives a reasonably good fitting and surprisingly enough, a simple central force (Y) worked better than most of them With Kuo's bare G-matrix (K,) the spectrum is not reproduced, justifying the inclusion of core polarization graphs which should compensate the effects of the space truncation.

On the other hand, it seems that the effective interaction obtained from the separable potential (KLS) contains most of the relevant effects, as a direct solution of the reaction matrix equations. The reduced matrix elements of the interactions indicated by (K,), (S) and (D) are equally bad from the point of view of this calculation.

Works on the independent pair model[19] have reached the conclusion that low-energy nuclear dynamics is mostly governed by the atractive well. This could justify the good agreement with experimental results obtained with a simple Yukawa force.

As for Si^{28}, only the renormalized two-body interaction of Kuo worked reasonably well. The ones which did not work well for Ne^{20} are still bad in this case and even the (KLS) and (Y) interactions gave a compressed spectrum.

The energy levels represented with dashed lines were calculated with the rotational expression $E_J = E_0 + A^{(n)} J(J + 1)$, where A = 1/(2 x moment *of* inertia) was taken from Table II. They are expressed in MeV. In Fig. 3, we have the energy spectrum of Ar^{36} determined exclusively by this procedure. Initially, the inertial paramater was calculated with the original Hamiltonian H using the expression of Thouless and Valatin (TV) and of Banerjee, de Oliveira and Stephenson (BDS), formulas (3-10) and (3-11), respectively; it was straightforward to proceed iteratively, as proposed in Ref. 4, to obtain a self-consistent value for A. Table II is divided into two parts: on the left, we have the results coming directly from H, that is, the first output in the iterative process which started with $A^{(0)} = 0$; on the right, there are values which appeared after three or four steps of iteration with the intrinsic Hamiltonian $H_i^{(n)} = H - A^{(n)} J^2$, as described in Section 3.

Both formulas (TV) and (BDS) converge simultaneously to a common $A^{(n)}$ value, and we can notice that this convergence value is closer to $A^{(0)}_{BDS}$. Also, it can be verified that the rotational levels do not differ very much when we use either $A^{(0)}_{BDS}$ or $A^{(0)}_{TV}$. Furthermore, these rotational spectra in dashed lines are close to those calculated with the Peierls-Yoccoz projection method. Although the techniques are apparently different, they are based upon the same self consistent (Hartree-Fock) description of the nucleus as a starting point. These results are indicative that using the HF approximation the various prescriptions to get the nuclear low-energy levels are almost equivalent. Recently, Friedman and Wilets[20] studied the "Formal aspects of nuclear moment of inertia theory" showing that the crancking model and the projection method give similar results for the moment of inertia (for low J and large (J^2)), provided that the cranking method is used self-consistently. Here, self-consistency means that we should take into account the two-body interaction terms in Eq. (3-8), which represent the change in the self-consistent field produced by the rotation. The importance of these terms was first pointed out by Thouless[2] in contraposition to the old Inglis formula. In Table II, we also display the values of the inertial parameter calculated with the Inglis formula (3-12).

Once the spectrum of energy is calculated, its agreement or not with the experimental data is a different matter. In our case, it depends (a) on how the HF wave function was calculated, more specifically on the fact that the single-particle space of states was restricted to levels between O^{16} and Ca^{40}, and (b) on the kind of two-body interaction used. In fact, we could expect[21] that by enlarging the harmonic oscillator space of single--particle states the calculated spectrum of energy would be improved. However, we did not have at hand the interaction matrix elements coupling different major shells. By doing a restricted Hartree-Fock calculation, we found the various spectra described in Figures 1, 2 and 3, suggesting that the renormalized matrix elements of Kuo (K) give the best fit to experiment.

In Tables III and IV, we show the electric-quadrupole reduced transition strengths in Weisskopf units (W.U.) for the ground state band of Ne^{20} and Si^{28}, respectively. For each type of two-body interaction, the calculation was performed with the HF wave functions determined with the Peierls-Yoccoz projection method. The value of 0.5 for the effective charge was chosen for both nuclei. The experimental data on Ne^{20} are those of Hausser *et al.*[22], and for Si^{28} we are comparing our theoretical results with the experimental figures given by M. M. Aleonard et *al.*[23]. All interactions seem to give very similar results so that the electric quadrupole transitions

may not be the ideal thing to look at in order to decide which is the best set of G_{JT} matrix elements. For instance, with **(K)** or (K_b), the results are practically the same and this could suggest that renormalization corrections might not be so important. E. C. Halbert et *al.*[24] did an extensive shell model calculation in the s-d shell. They also found that the $B(E2)$ values within the ground state bands are essentially the same, using different interactions. Their value for the $2^+ \rightarrow 0^+$ transition in Ne^{20} is **15.0 W.U.** with Kuo's renormalized interaction (K), and effective charge equal to **0.5.** For Ne^{20}, we also observe that the strength of the $6^+ \rightarrow 4^+$ transition is weaker than that of the $4^+ \rightarrow 2^+$ transition. This has been verified in all our cases, in good agreement with experiment. In spite of the calculated transitions being too close to each other, we could still say for Ne^{20} that the (K) interaction gives the most favorable results. They also agree well with the SU_3 calculation of M. Harvey[25].

In the case of Si^{28} the results are practically the same for all interactions used and too far from the experimental data The SU_3 values obtained by Harvey[25] are **31.95 W.U.** for the $2^+ \rightarrow 0^+$ transition and **43.10 W.U.** for the $4^+ \rightarrow 2^+$ transition. In our calculation with the Hartree-Fock projected wave functions, the values are half-way between the experimental data and those obtained in the SU_3 scheme. In the adiabatic rotational model, the reduced transition probability is proportional to the square of the Clebsh-Gordan coefficient $(J_i 020 \mid J_f 0)$, for transitions within the lowest $K = 0$ band. The resulting values will increase with the angular momentum of the states involved and this is not what happens in the SU_3 or in the Peierls-Yoccoz projection method.

For the study of excited states, we solved the RPA equation **(4-9)** to obtain the energy $\hbar\omega_\lambda$ of each mode of excitation, for the vanous band quantum number **K.** Each excited state is also characterized by the isotopic spin quantum number T. Considering that the G_{JT} matrix elements **(K)** (KLS) and (Y) were the ones that gave better energy levels for the ground state band of Ne^{20}, we reproduce in Table V the RPA results corresponding to these two-body interactions. The energies are in MeV.

The Tamm-Dancoff approximation treatment of the nuclear excited states, practically gives the same energies, as it has been previously verified[26]. The only difference occurs for the first state with band quantum number K = **1,** whích is spurious. One advantage of the RPA treatment is to make the separation of this spurious state possible; it comes out with zero energy.

To illustrate this point, we show in Table V the Tamm-Dancoff results obtained with the (KLS) interaction. This situation is common to all three nuclei with any interaction that we have used.

From a given collection of experimental energy levels with the same J value, not belonging to the ground state band, we picked out the one with the lowest energy and compared it with our lowest theoretical excited energy level of the same $K = J$ band The experimental energies were taken from Ref. 27. Evidence for the existence of a $J^{\pi} = 1^{+}$ level at 9.95 MeV was given in Ref. 22. There is, of course, some arbitrariness in the choice of these experimental energy levels as one does not know whether they really are band heads.

In the case of Ne^{20}, we have a reasonably good agreement with experiments for all three types of two-body interactions. The only large discrepancy occurs for the first 0^{+} excited level obtained with the Yukawa (Y) force as being at 4.16 MeV when the first 0^{+} level is really at 6.72 MeV. Again, the best fitting seems to occur for Kuo's (K) renormalized two-body matrix elements.

On the other hand, in the cases of $Si^{2'}$ and Ar^{36} the comparison with the available data is equally poor for all types of interactions. In Tables VI and VII, we display the results obtained with the (K) interaction, only.

The correlations which are present in the RPA ground state, modifying the previous Hartree-Fock description, are measured as the energy difference between the two ground states according to Eq. (4-14). We show in Table VIII these correlation energies for Ne^{20}, $Si^{2'}$ and Ar^{36}. These values are of the same order of magnitude for all three nuclei, but $Si^{2'}$ shows more correlations in its GS.

In most cases, the effective interactions denved from the realistic two-body potentials are constructed in a convenient configuration space of two--particles. After obtaining a set of two-body matrix elements in this case, it is assumed that they are independent of the number of valence nucleons and one uses them to calculate the properties of nuclei having more than two nucleons outside the core. This is a usual procedure in standard sheil model treatments[24].

The sets of G_{JT} matrix elements that we used were determined for the A = 18 nuclei (A = mass number).

As the number of particles outside the O^{16} core increases, the calculations involving the use of these same matrix elements should become less accurate. In our case, the results for Ne^{20} are better than for Si^{28} and Ar^{36}, when we take as G_{JT} the sets (K), (KLS) or (Y). Concerning these last two nuclei, although the comparison with the experimental data has become worse, the renormalized effective interaction of Kuo (K) still gives more reasonable results.

All results we found depended also on the fact that our calculation involved only one major shell; if possible, in order to make the treatment more general, the Hartree-Fock calculation should be performed with major shell mixing.

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