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Phase Shifts for K^+-Proton Scattering with Strong p-Waves at Low Energies*

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A soiution for the phase shift analysis of K^+ p scattenng at low energies which is charactenzed by large p-wave and small s-wave phase shifts is presented and discussed.

Apresenta-se e discute-se uma solução para a análise de defasagens no espalhamento K^+ p a baixas energias que é caracterizada por grandes defasagens da onda p e pequenas da onda *s*.

1. Introduction

The general feature of K^+-proton scattering at low energies (up to 800 MeV/c for meson momentum in laboratory system) is the absence of a marked structure. The data for total and differential cross-sections obtained by Goldhaber et al.[1], Focardi et al.[2], Stubbs et al.[3] and Kycia et al.[4] were compiled by the Particle Data Group[5]. The inelastic channels are open for momenta larger than 525 MeV/c, but only above 870 MeV/c (when N* production starts) do they become really important. In this paper, we are interested in energies up to about 800 MeV/c, so that we ignore inelastic channels and take all phase shifts as real.

The experimental work with the K^+p system has been more extensive for momenta larger than 1 GeV/c. The data at low energies are rather scarce and insecure. Polarization data are missing almost completely so that the final results of a phase shift analysis cannot be obtained at the present time.

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The data for momenta up to 800 MeV/c are usually described in terms of a large s-wave phase shift with very small contributions coming from other waves [1,2,6,7]. These analyses of the data have been made with s-wave dominance taken as an assumption since no information is available on the polarization of the recoiling proton to allow for a selection between this and other possibilities.

The assumption of larger s-wave phase shift is justified with the well-known argument that, for short-range interactions, low l-waves are more effective in the scattering process and with the fact that the angular distribution seen in K^+ p scattering does not show strong deviations from isotropy. Of course, this is a sensible attitude. However, we think that other possible solutions in the partial-wave analysis must also be considered since they may in the end come out to be the correct ones.

We would like to mention an example in which s-waves are suppressed at low energies and p-waves dominate the scattering process. Consider a velocity-dependent potential of the form

$$V = -(\lambda/2m)\mathbf{p} \cdot J(r)\mathbf{p} \tag{1}$$

in Schrodinger equation. p is the momentum operator, m is the mass, λ is a dimensionless coupling constant, and $J(r)$ is a function of the distance to the origin such that $J(r) = 1$ for $r < b$ and $J(r) = 0$ for $r > b$ (square well or barrier). It has been shown[8] that for such a problem the s-wave phase shifts behave like k^5 at low energies while for the other waves the usual k^{2l+1} dependence holds. These velocity-dependent potentials have a sensible form, and do not seem to be particularly unrealistic (as far as potentials are concerned) as they have been used rather successfully in the fitting of nucleon-nucleon scattering data In Fig. 1, we show the energy dependence of the $l = 0$ and $l = 1$ phase shifts calculated with the potential given in Eq. (1). The value $\lambda = -10$, chosen for the coupling constant, corresponds to a repulsive interaction.

In the present paper, we wish to present a solution for the low energy K^+p phase shift analysis which is characterized by large $p_{3/2}$ and $p_{1/2}$, and small s-wave phase shifts. Our analysis shows a smooth variation of the parameters with the energy, and covers the available data from 140 MeV/c to 864 MeV/c. We do not claim that the solution here discussed is more acceptable than the conventional one, but we do not find a strong reason why it should be discarded from the start.

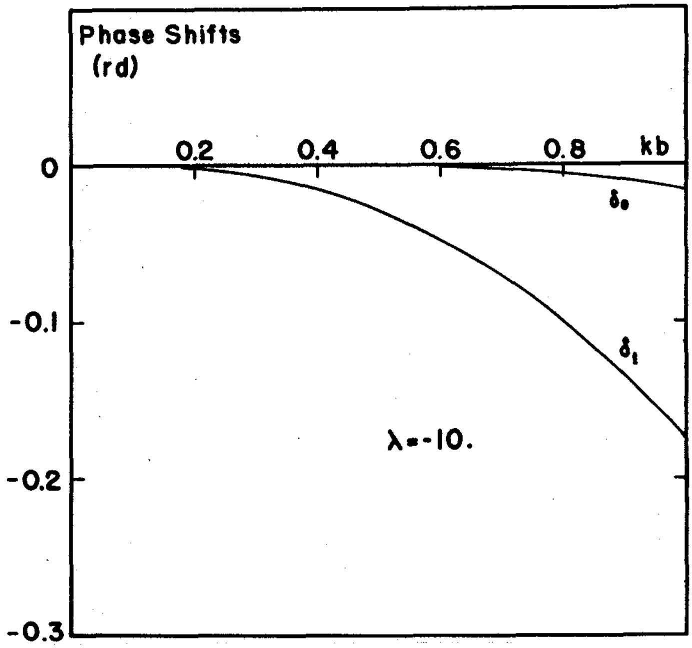

Fig. 1 - Dominance of p-waves at low energies obtained in a potential scattering model. The velocity-dependent potential $V = -(\lambda/2m)\mathbf{p}\,.\,J(r)\mathbf{p}$, with $J(r) = 1 - \theta\,(r-b)$ leads to an s-wave phase shift δ_0 which behaves like k^5 while the p-wave phase shift δ_1 goes like k^3.

The solution here presented shows a large repulsive $p_{3/2}$ and a smaller attractive $p_{1/2}$ phase shift and an almost negligible s-wave contribution. Both p-wave phase shifts vary with the energy in a way compatible with the expected k^3 dependence at low momenta.

2. Fitting of the Data

The available experimental data on differential cross-sections for momenta equal to 140, 175, 205, 235, 265; 355, 520, 642, 778, 810 and 864 MeV/c were fitted with the usual formula for the differential cross-section in terms of partial-wave amplitudes. Only s- and p-waves were considered and the Coulomb interaction was taken into account in the usual way.

q (MeV/c)	k (f^{-1})	A (f)	B (f^3)	C (f^3)	N	χ^2	$\chi^2/(N-3)$	Confidence Level
140	0.46	0.0080	−1.0570	0.3617	6	6.28	2.09	0.09
175	0.58	−0.0320	−0.7356	0.2230	6	1.31	0.44	0.80
205	0.67	0.0425	−0.4900	0.1770	6	7.59	2.53	0.15
235	0.78	0.0316	−0.3551	0.1321	6	8.61	2.87	0.04
265	0.88	−0.0022	−0.2703	0.1160	6	3.51	1.17	0.33
355	1.18	−0.0220	−0.1525	0.0765	5	2.91	0.97	0.22
520	1.72	0.0000	−0.0740	0.0450	5	0.84	0.42	0.80
642	2.13	0.0061	−0.0514	0.0248	5	0.43	0.21	0.90
778	2.58	0.0034	−0.0361	0.0197	20	15.15	0.89	0.60
810	2.68	0.0135	−0.0340	0.0187	7	1.47	0.34	0.90
864	2.84	0.0122	−0.0320	0.0132	20	27.10	1.59	0.08

Table I — Values of the parameters A, B, C obtained in fitting the experimental data N is the number of points in the data for the differential cross-section at each energy, and N − 3 is the number of degrees of freedom. We show also the values obtained for χ^2 and the corresponding probabilities; q is the incident meson momentum in the laboratory system and k is the centre-of-mass momentum.

Let us write partial-wave amplitudes in the form

$$
\begin{aligned}
f_0 &= A/(1 - ikA), \\
f_1^{(+)} &= Bk^2/(1 - ik^3B), \\
f_1^{(-)} &= Ck^2/(1 - ik^3C),
\end{aligned}
\tag{2}
$$

where k is the momentum in the centre-of-mass system. With k measured in fermi^{-1}, the parameters A, B, C are expressed in fermi, fermi^3 and fermi^3, respectively. They were determined for each of the energies listed above. For all energies, we were able to find a solution for the fitting process characterized by a small value for A and large values of B and C, The results are shown in Table I. The value of A is compatible with zero at some energies, while B and C vary smoothly with the energy, as shown in Fig. 2.

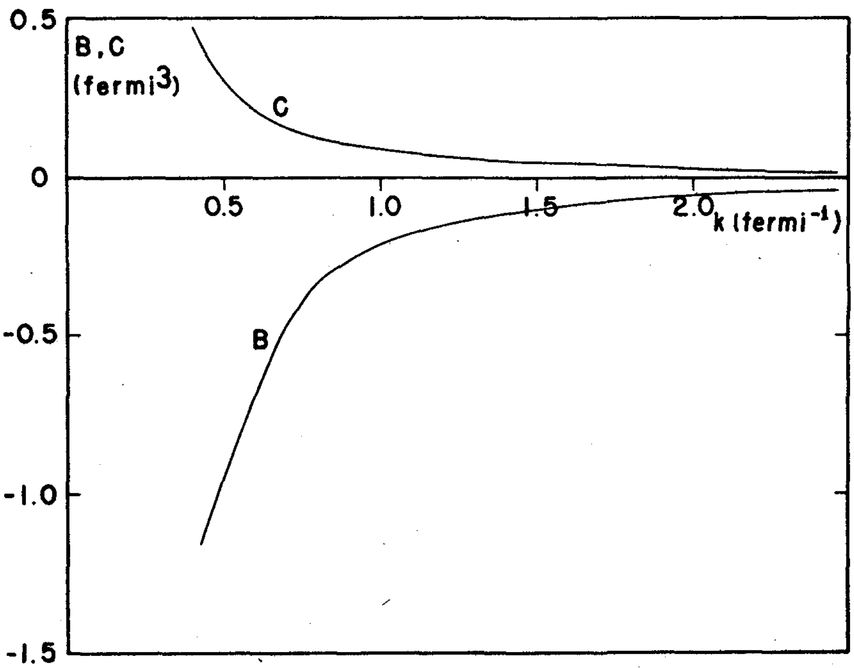

Fig. 2 - The values obtained for the parameter *B* and *C* vary smoothly with the centre-of-mass momentum *k*. Both *B* and C go to infinity for small k in such a way that k^3B and k^3C remain finite.

In the absence of Coulomb interaction, $d\sigma/d\Omega$ is invariant under a simultaneous change of sign of all three parameters. The sign was determined

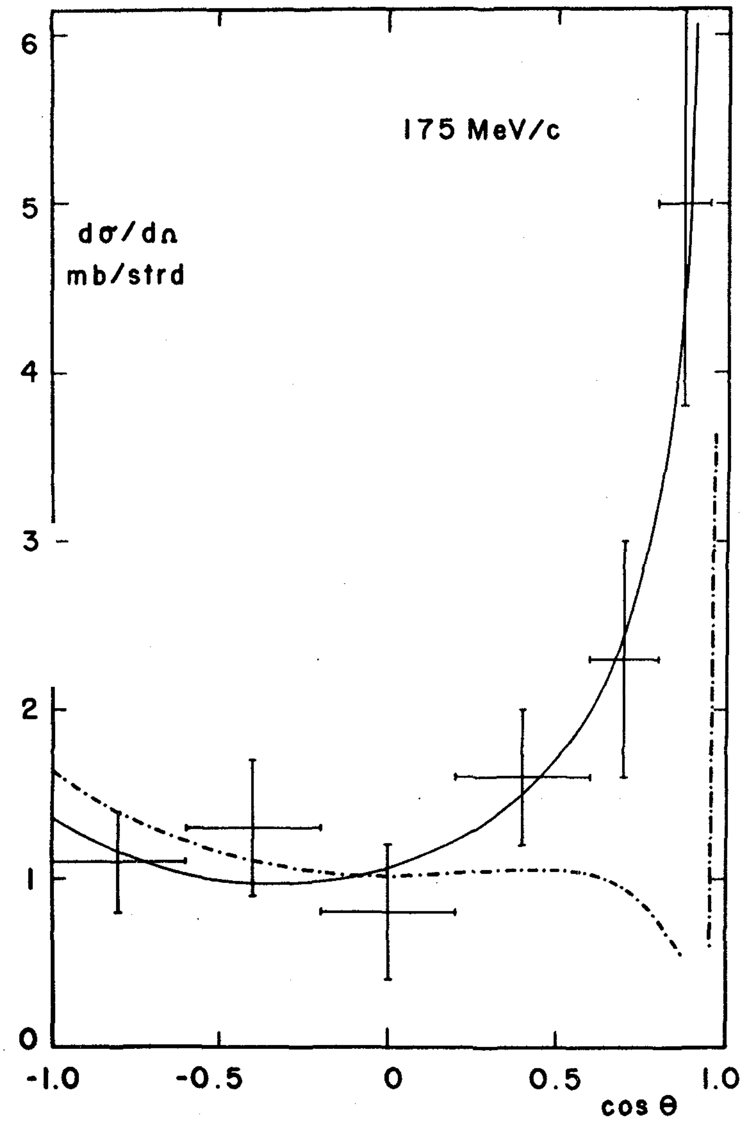

Fig. 3 - Fitting of the differential cross-section ai 175 MeV/c, showing the sign of the interference with the Coulomb interaction. The solid line shows ihe curve obtained with the values of the parameters as given in Table I. The dashed line shows the curve obtained by inverting the sign of the three parameters The interference with the Coulomb interaction is constructive, indicating a dominance of the repulsive nature of the interaction.

by considering one particular energy in which the Coulomb interference effects play an important role in the fitting. Fig. 3 shows in solid line the best fitting curve, with the values for the parameters given in Table I, for incident momentum q = 175 MeV/c. In dashed line, we show the curve obtained by inverting the sign of the three parameters. The figure shows that the interference with the Coulomb interaction is constructive and that the interaction as a whole has a repulsive nature[1].

In Figs. 4a – 4j, we show the data, at the other energies, fitted with values of A, B and C as given in Table I.

Fig. 4a-j - Fitting of the data for the differential cross-sections. The values of the parameters which produce the curve are given in Table I.

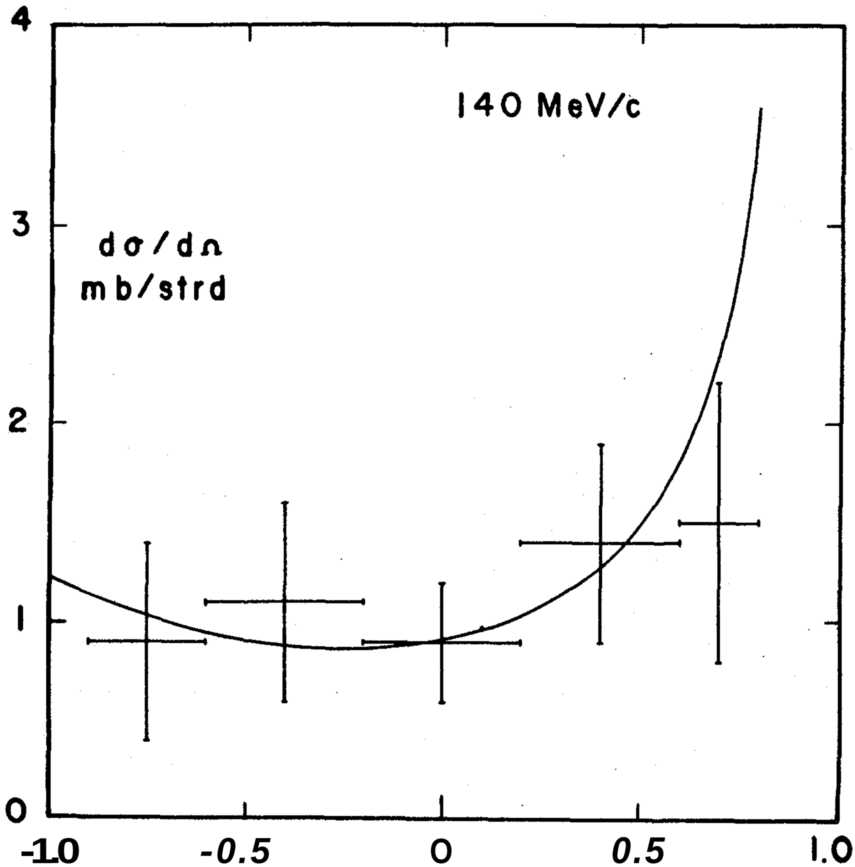

Fig. 4a

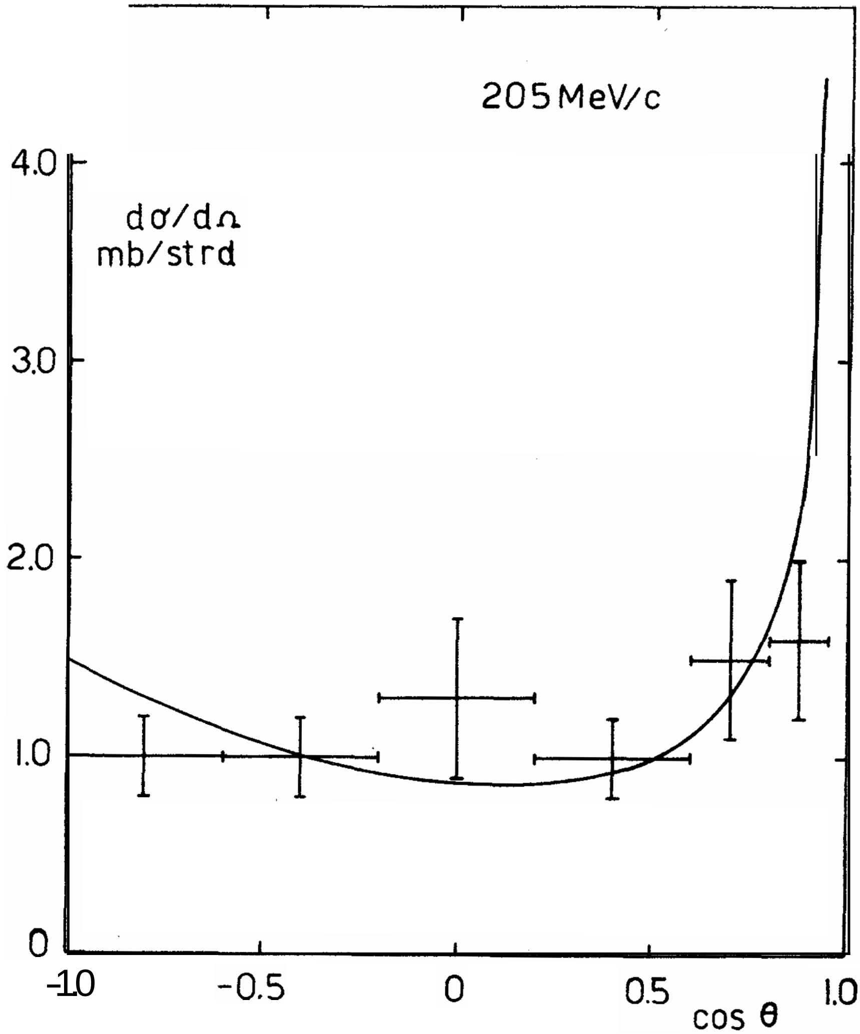

Fig. 4b

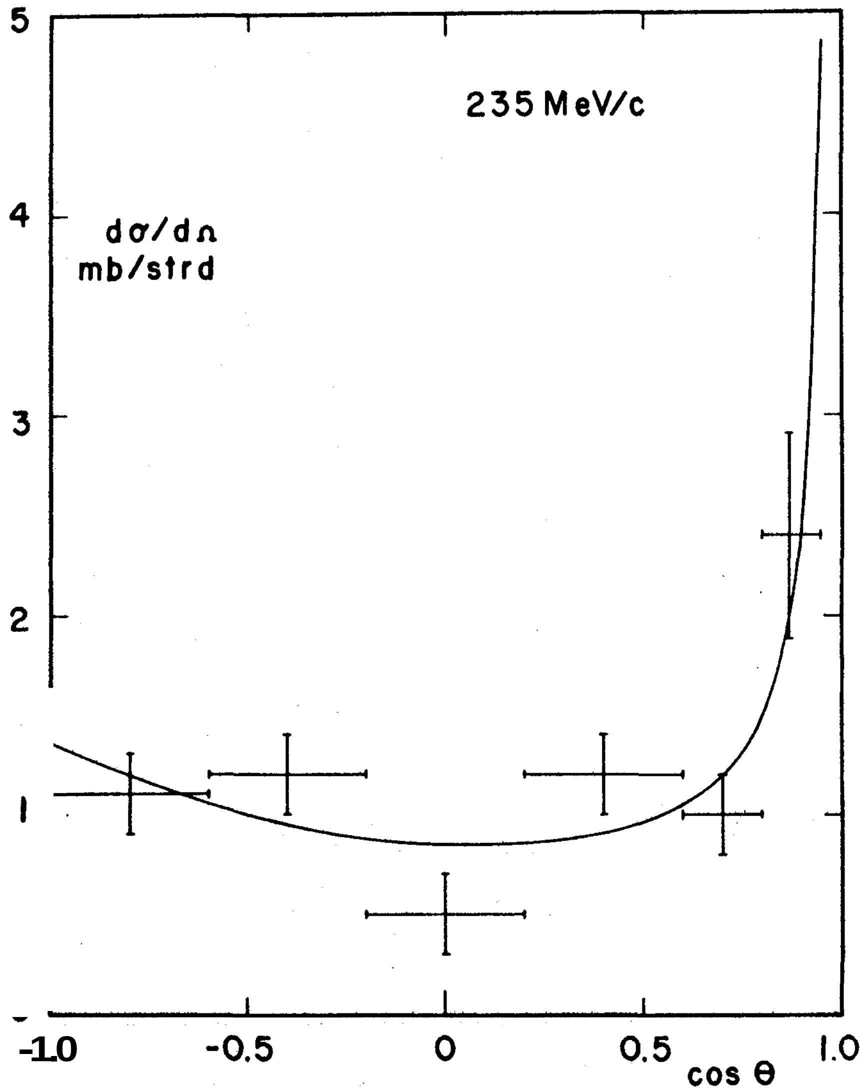

Fig. 4c

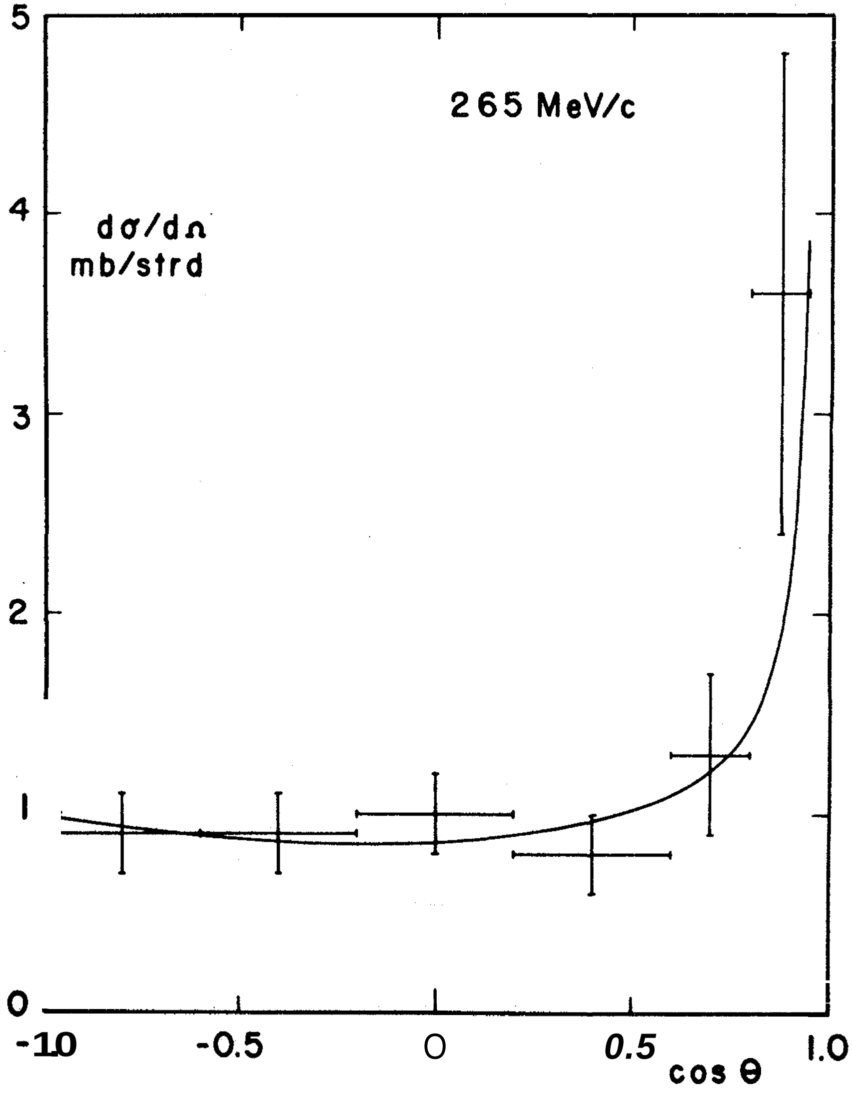

Fig. 4d

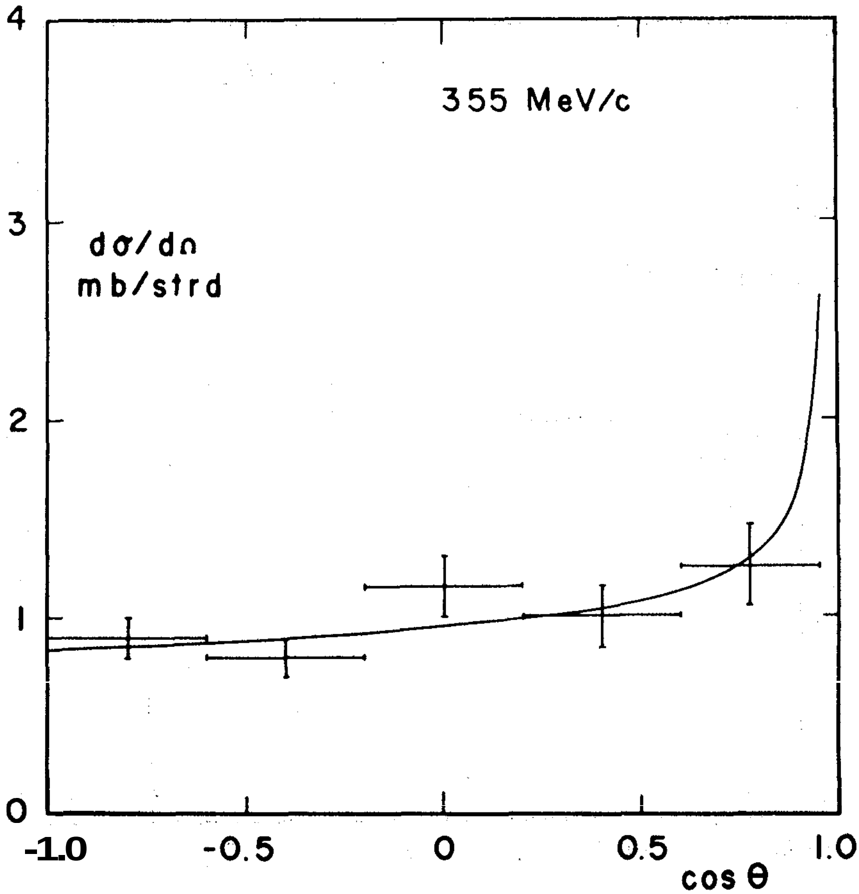

Fig. 4e

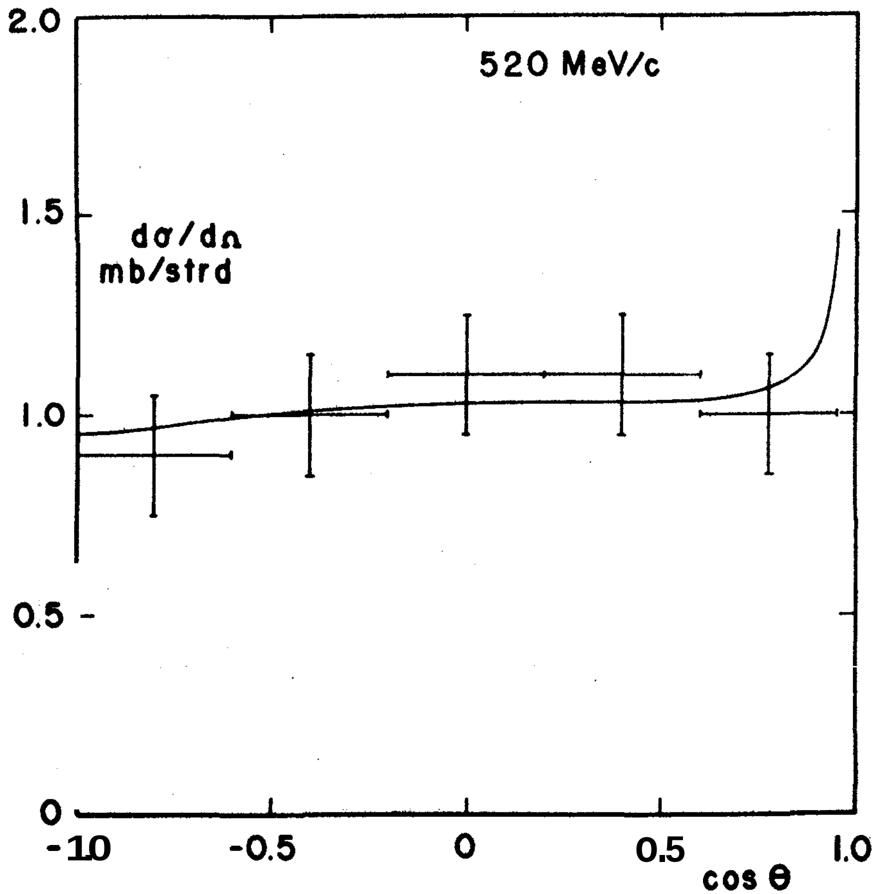

Fig. 4f

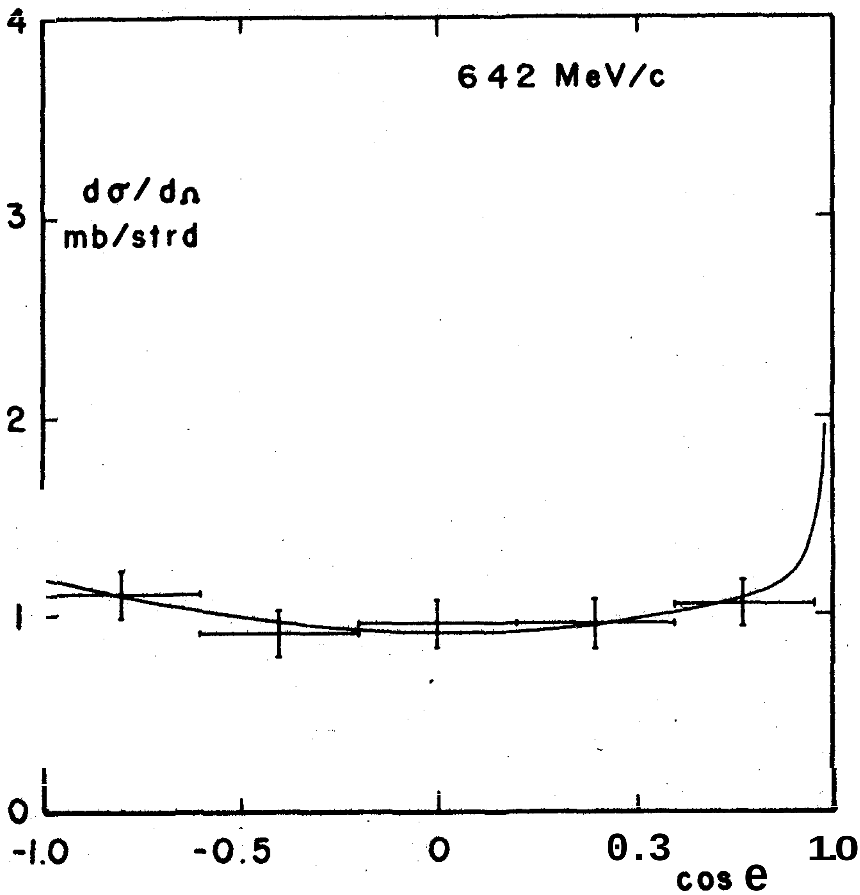

Fig. 4g

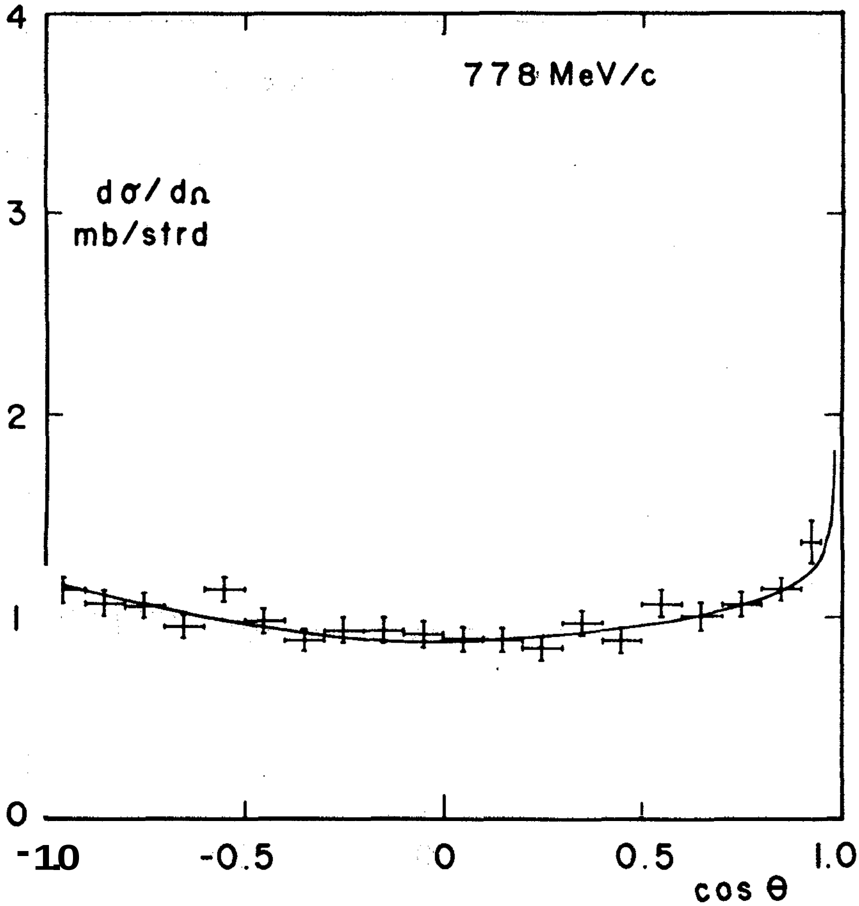

Fig. 4h

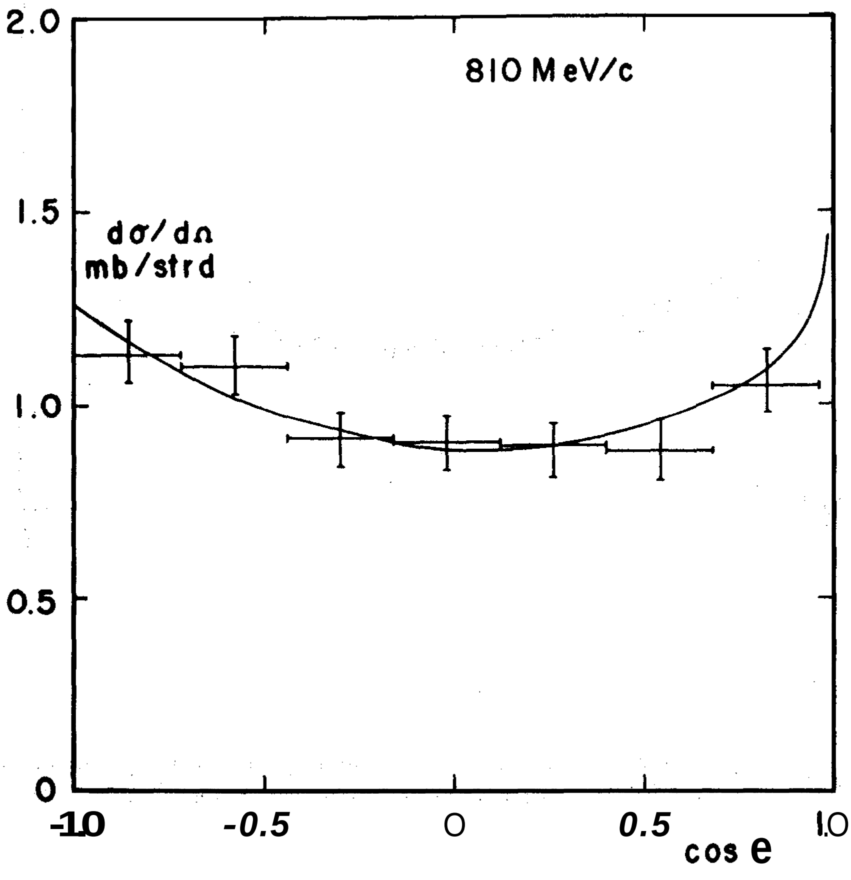

Fig. Si

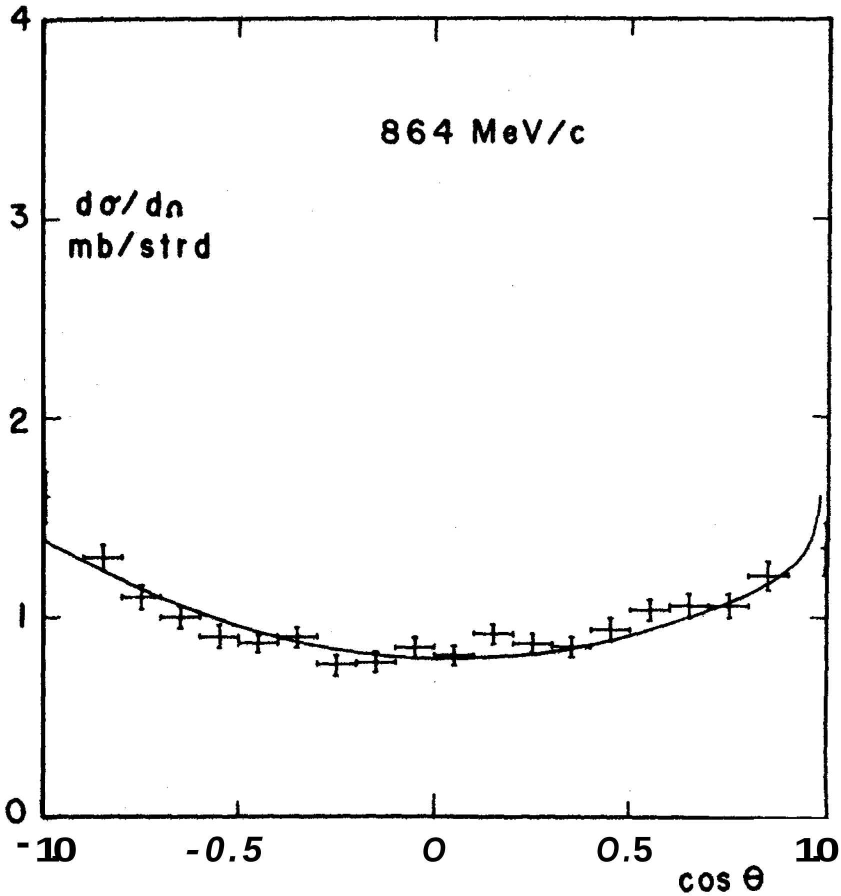

Fig. 4j

3. Phase Shifts and Conclusions

We call δ_0 the s-wave phase shift and δ_1 and δ_3 the p-wave phase shifts for $j = 1/2$ and $j = 3/2$, respectively. The relations between A, *B*, C and the phase shifts are

$$\tan\,\delta_0 = kA, \qquad \tan\,\delta_{3/2} = k^3B, \qquad \tan\,\delta_{1/2} = k^3C. \tag{3}$$

The values obtained for the phase shifts are shown in Table II. We see a dominating negative $p_{3/2}$ phase shift, a smaller and positive δ_1, and an almost negligible s-wave contribution.

q (MeV/c)	k (fermi^{-1})	δ_0 (deg.)	δ_1 (deg.)	δ_3 (deg.)
140	0.46	0.2	2.0	−5.9
175	0.58	−1.1	2.5	−8.2
205	0.67	1.6	3.0	−8.4
235	0.78	1.4	3.6	−9.6
265	0.88	−0.1	4.5	−10.4
355	1.18	−1.5	7.2	−14.1
520	1.72	0.0	12.9	−20.6
642	2.13	0.7	13.5	−26.4
778	2.58	0.5	18.7	−31.8
810	2.68	2.1	19.8	−33.2
864	2.84	2.0	16.8	−36.2

Table II - Solution for the phase shift analysis with dominance of p-waves. The s-wave phase shift δ_0 is compatible with zero. 6, and 6, are the p-wave phase shifts for $j = 1/2$ and $j = 3/2$, respectively.

It is certainly an important point that the phase shifts present a smooth dependence with the energy. This has not been imposed as condition to be satisfied in a systematic way, but the solutions obtained for the phase shifts do in fact present the desired smoothness. In Fig. 5, we show the values of δ_1 and δ_3 plotted against the centre-of-mass momentum.

Another important point is that *p*-wave phase shifts are expected to behave at low energies like the third power of k. To show that our solutions are compatible with this condition, we have applied a best fitting method to find the values of the parameters a_i and b_i such that

$$k^3 \cot \delta_i = \frac{1}{a_i} + b_i k^2, \tag{4}$$

where i = 1,3. In this fitting, the values of δ_1 and δ_3 from 140 to 642 MeV/c were used. The values obtained for the parameters are shown in Table III.

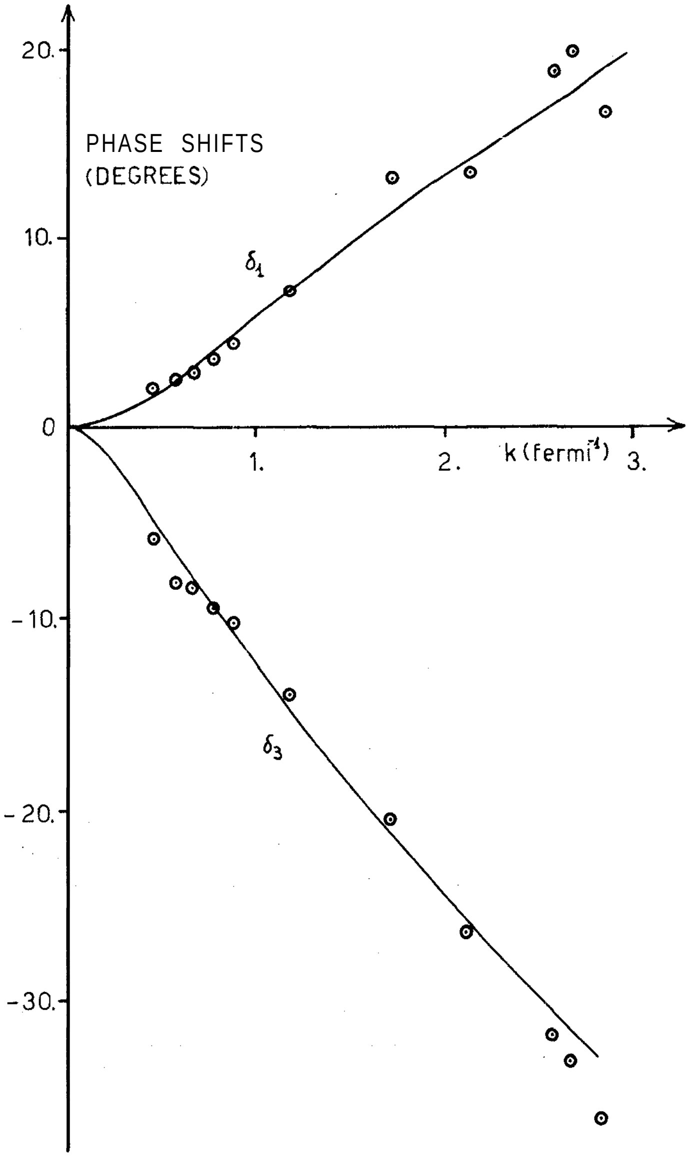

Fig. 5 - Values obtained for the p-wave phase shifts. The curves were drawn using the values of δ_1 and δ_3 obtained from Eq.(4), with values of the parameters as in Fig. 6 and Table III.

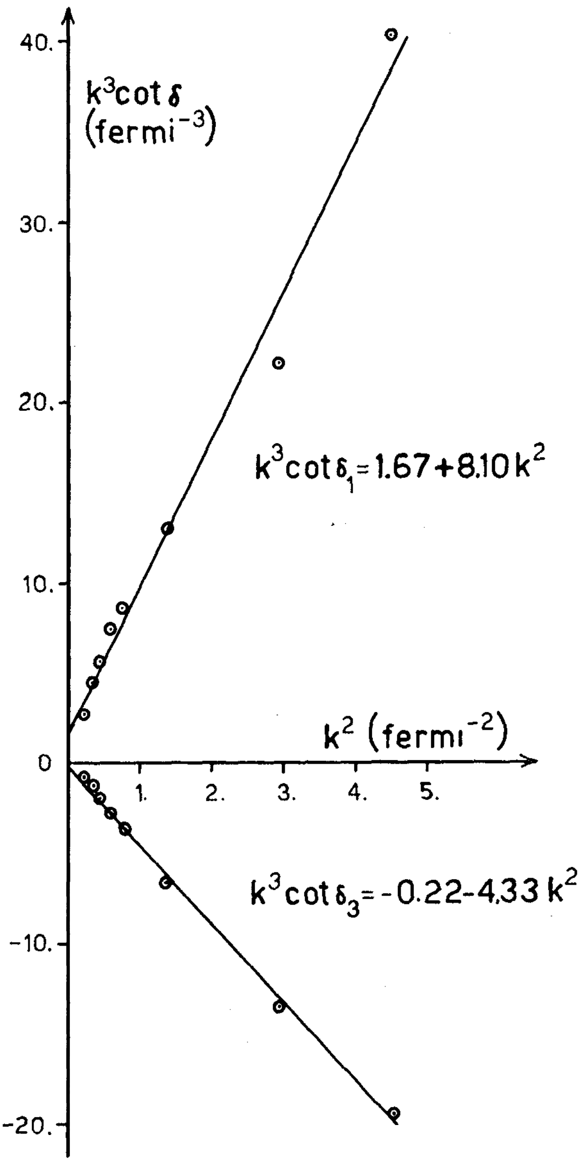

Fig. *6* - Fitting of the p-wave phase shifts with the formula $k^3 \cot \delta_i = -1/a_i + b_i k^2$.The eight points corresponding to incident momenta from 140 to 642 MeV/c were used to find the best values of a_i and b_i.

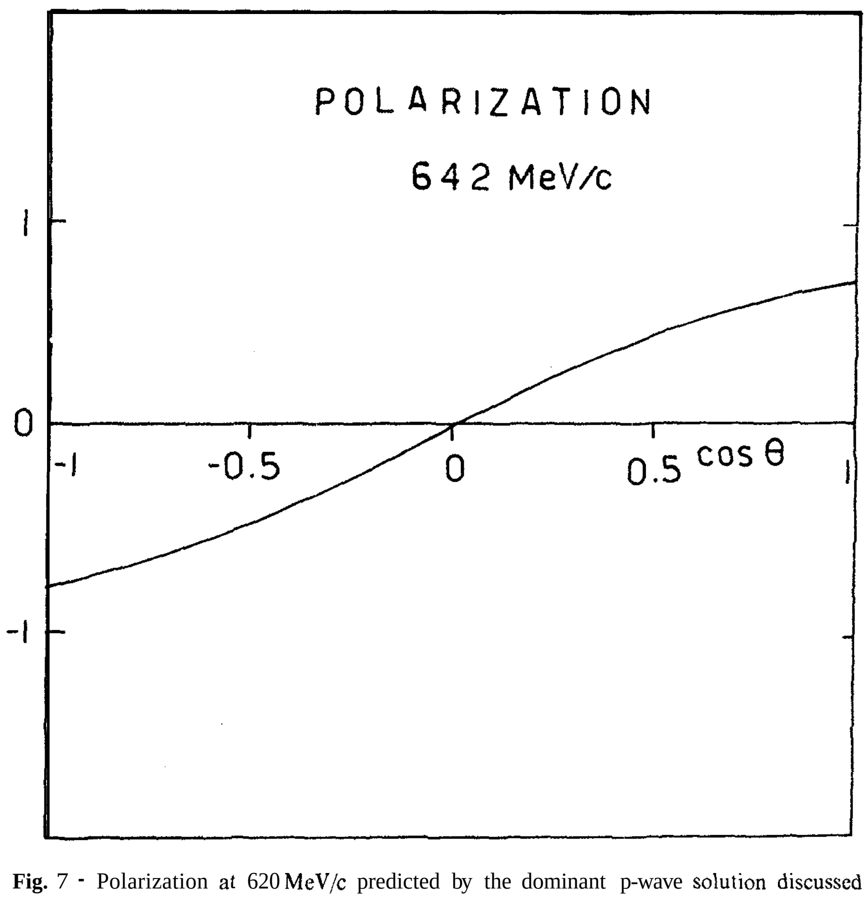

Fig. 7 - Polarization at 620 MeV/c predicted by the dominant p-wave solution discussed in this paper. The sign of the polarization *is* that of k_{in} x k_{out}, *according* to the *accepted* convention. **A** factor $\sin\theta$ has been separated from the formula for the polarization vecior so that the curves do not go to zero in the extreme angles.

	a_i (fermi^3)	b_i (fermi^{-1})	No. of Points	χ^2
$i = 1$	0.6	8.10	8	0.90
$i = 3$	−4.5	−4.33	8	0.14

Table III - Values of the parameters in Eq. (4) which give the best fitting for $k^3 cot\,\delta_i$ from 140 to 642 MeV/c.

In Fig. 6, we give a graphical representation of these results. It seems beyond doubt that the solution here presented is compatible with the required k behaviour. This shows the restricted validity of one of the arguments that have been used for the choice of the s-wave dominant set of phase shifts, namely, that the experimental results show that the phase shifts behave at low energies as a first power of k[1,6]. Further experimental data at the lowest energies are needed to decide this question.

The data on the polarization of the recoilling proton which are necessary for the unique determination of phase shifts are not yet available at low energies. Some information[9] has been obtained for momenta in the region above 865 MeV/c. Measurements[10] were also made at 780 MeV/c but they do not seem to agree with those made at the neighbouring higher energies. Polarization experiments are necessary at energies below the effective opening of the inelastic channels so that the phase shift analysis for the K^+p system can be performed. In Fig. 7, we show the polarization at 620 MeV/c predicted by the dominant p-wave solution presented in this paper.

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