Distortion in Quasi-Free Scattering

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Distorted momentum distributions of (p, $2p$) reactions in ^{16}O are calculated with a complex distorting potential consistent with the nucleon density distribution in the nucleus. A comparison with conventional calculations using an equivalent but "non-consistent" potential shows considerable differences, in particular for the distorted momentum distribution of the $1s$ protons. Some more general aspects, important for the detailed interpretation of quasi-free experiments, are pointed out and briefly discussed.

Distribuições de momentum distorcidas obtidas de reações (p, $2p$) em ^{16}O são calculadas com um potencial de distorção complexo consistente com a distribuição de densidade dos núcleons no núcleo. Uma comparação com cálculos convencionais em que se utiliza um potencial ótico equivalente mas "não consistente" mostra diferenças consideráveis, em particular para a distribuição de momentum distorcida dos prótons $1s$. Alguns aspectos mais gerais, importantes para a interpretação detalhada de experiências quase-livres, são salientados e discutidos resumidamente.

1. Introduction

Quasi-free (p, $2p$) and (e, e'p) scattering experiments have clearly demonstrated the existence of "inner" nuclear shells, i.e., the $1s$ shell in $1p$ nuclei and the $1s$ and $1p$ shells in 2s-ld nuclei. At present such experiments constitute almost the only source of information on the energy and momentum distributions of the single hole states in these shells. Reviews of this field are given in Ref. 1; for later experimental results see, e.g., Refs. 2-7.

In the theoretical analysis of quasi-free experiments one calculates the energy and angular dependence of the coincidence rate of the two emerging particles on the basis of a nuclear model. In essence the angular correlation is given by the momenturn distribution of the overlap integral of the relevant states of the initial and final nuclei, distorted by the initial and final state interactions of the incoming and of the two outgoing particles. This distortion is not essential in the case of (e, e'p) scattering at

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300-1000 MeV, but it turns out to be quite important in the analysis of $(p, 2p)$ experiments. In this case the intensity may easily be reduced by orders of magnitude and also the shape of the distribution curves is in general seriously modified. It is therefore clear that for the detailed interpretation of the $(p, 2p)$ experiments these initial and final state interactions should be taken into account as well as possible; in general for such calculations a complex distorting potential is used. The first distorted wave calculation of this type was performed for the case of ^7Li in Ref. 8; subsequent calculations have followed the same line. In most cases the semi-classical approximation has been used.

One general drawback in all these calculations is that no care has been taken to match the nuclear wave functions with the distorting optical potential, i.e., the density distribution used to obtain the complex potential has not been derived from the nuclear wave function as it should. In Ref. 8, for example, single particle harmonic oscillator wave functions are used in the overlap integral, whereas a square well optical potential is chosen. In Ref. 9 on the other hand, exponential and harmonic oscillator wave functions are employed, whereas the distorting potential is Gaussian in shape. A similar situation is met in all other calculations.

That this lack of matching of the wave functions with the distorting potential may be quite serious for $(p, 2p)$ calculations is clear because of the following. In the actual case of a strongly absorbing potential, the parts of the wave functions which extend to regions where the potential is small will give the main contribution to the quasi-free cross section; this non-overlap is, of course, strongly dependent on the relative shapes of the wave functions and the distorting potential.

The purpose of the present paper is to investigate this effect. We take for simplicity a light nucleus, namely ^{16}O, and generate the nuclear wave functions from a given shell model potential; we then take the distorting optical potential proportional to the nuclear density as calculated from these wave functions and compute the angular correlation. This "consistent" way of taking the distortion into account may then be compared with a calculation using the same parameters but a conventional shape for the distorting potential.

Such a comparison shows that already for a nucleus as light as ^{16}O the above mentioned effect is significant both with respect to the size and to the shape of the angular correlation cross section. For somewhat heavier nuclei the effect is likely to be even more pronounced.

2. Calculation of Cross Section

In this section we review the calculational procedure used to determine the distorted momentum distribution, and give the method employed to obtain the necessary parameters and the results for the distorted momentum distributions.

The cross section for a coplanar symmetric (equal energies and angles for the outgoing protons) (p, 2p) process in the distorted impulse approximation is given in the usual notation[1] by

$$\frac{d\sigma}{dE_1\, d\Omega_1\, dE_2\, d\Omega_2} =$$

$$= \frac{4}{\hbar^2 c^2} \frac{k^2}{k_0^2} \frac{\hbar^2 c^2 k^2 \sin^2\theta + M^2 c^4}{\hbar^2 c^2 q^2 + M^2 c^4} \frac{d\sigma^{fr}}{d\bar{\Omega}} P(\mathbf{q})\, \delta(2E + E_{A-1} - E_0 - E_A). \qquad (1)$$

For a single-hole model, without spin-orbit splitting and the distortion being taken into account in a semi-classical approximation, the distorted momentum distribution $P(\mathbf{q})$ is given by

$$P(\mathbf{q}) = \frac{N_l}{2l+1} \sum_m |g_l'^m(\mathbf{q})|^2 \qquad (2)$$

with

$$g_l'^m(\mathbf{q}) = (2\pi)^{-3/2} \int exp\,(-i\mathbf{q}\,.\,\mathbf{r})\, \psi_l^m(\mathbf{r})\, D_0(\mathbf{r})\, D_1(\mathbf{r})\, D_2(\mathbf{r})\, d^3 r, \qquad (3)$$

$\mathbf{q} = 2\mathbf{k} \cos\theta - \mathbf{k}_0$ being the momentum of the nuclear hole produced in the symmetric (p, 2p) proces. In an extreme single-particle model, q is also the momentum the nuclear proton had in the nucleus before being knocked out and $\psi_l^m(\mathbf{r})$ is the wave function of the knocked-out proton, l and m being the orbital angular momentum and magnetic quantum number of this proton. The number of protons in the shell l is denoted by N_l and

$$D_0(\mathbf{r}) = exp\left\{-i\frac{E_0}{\hbar^2 c^2 k_0} \int_{-\infty}^{\mathbf{r}} V_0(\mathbf{r})\, ds_0\right\}, \qquad (4a)$$

$$D_j(\mathbf{r}) = exp\left\{-i\frac{E}{\hbar^2 c^2 k} \int_{\mathbf{r}}^{\infty} V_j(\mathbf{r}')\, ds_j\right\} \quad (j = 1,2), \qquad (4b)$$

are the distorting factors of the incoming and the two outgoing ($j = 1,2$) protons with energy-momentum four-vectors $(E_0/c, \hbar\mathbf{k}_0)$ and $(E/c, \hbar\mathbf{k}_j)$,

$|\mathbf{k}_j| = k$, respectively; the integrations in Eqs. (4) are to be performed over the classical paths of the particles.

Following Refs. 10, 11, we express the complex potentials V_0 and $V_1(= V_2)$ in the averaged forward nucleon-nucleon scattering amplitudes $\bar{A}_0$ and $\bar{A}_1(= \bar{A}_2)$:

$$V_n(\mathbf{r}) = -4\pi \frac{\hbar^2 c^2}{E} \bar{A}_n(0)\, \rho\,(\mathbf{r}), \qquad n = 0,1,2. \tag{5}$$

In this equation $\rho(\mathbf{r})$ is the nucleon density, with $\int \rho(\mathbf{r})\, d^3 r = 4\pi \int_0^\infty r^2 \rho(r)\, dr =$ $= A - 1$ (because the interaction of the knocked out nucleon has already been taken into account). One has thus for the distorting factors

$$D_0(\mathbf{r}) = exp\left\{\frac{4\pi i}{k_0} \bar{A}_0(0) \int_{-\infty}^{r} \rho(r')\, ds_0\right\} \tag{6a}$$

and

$$D_j(\mathbf{r}) = exp\left\{\frac{4\pi i}{k} \bar{A}_j(0) \int_{r}^{\infty} \rho(r')\, ds_j\right\} \cdot \tag{6b}$$

The above procedure is the standard one; at this point one usually chooses somewhat arbitrarily a reasonable density function $\rho(\mathbf{r})$ in Eq. (5) for the the distorting potential.

The main point in the present calculation is to use in Eq. (5) the nuclear density as obtained from the nuclear single-particle wave-functions $\psi_l^m(\mathbf{r})$. For simplicity we take the same distorting potential for the $1s$ and $1p$ knock-out processes and choose the density to be

$$\rho(r) = \frac{15}{16} \Sigma \{|\psi_{1s}(\mathbf{r})|^2 + |\psi_{1p}(\mathbf{r})|^2\} = \frac{15}{16} \frac{1}{4\pi} \{4R_{1s}^2(r) + 12R_{1p}^2(r)\}, \tag{7}$$

$R_{1s}(r)$ and $R_{1p}(r)$ being the $1s$ and $1p$ radial parts of the wave-functions, respectively.

With the density (7), the distorted wave calculation is performed for wave- -functions generated by a square-well potential. The constants of these wave-functions have been determined using the separation energies of the 1s and 1p protons as obtained in quasi-free experiments[12], namely 38 MeV and 17 MeV (the latter one being a weighted average over the $1p_{3/2}$ and $1p_{1/2}$ separation energies), and taking a value of 2.64 fm for the root mean square radius[13] of ^{16}O. Two values for the depth of the square--well potential were necessary to fit the separation energies, namely 50 MeV and 40 MeV for the 1s and 1p states, respectively, and the radius of the potential turned out to be 3.5 fm.

For the incident proton an energy of 170 MeV was taken and for the two outgoing protons equal (varying) angles and energies calculated from kinematics, were used. On the other hand, the real and imaginary averaged nucleon-nucleon forward scattering amplitudes have been taken always at 170 MeV (incoming protons), 76 MeV (outgoing protons from the 1p shell) and 66 MeV (outgoing protons from the 1s shell) by interpolating the results of Ref. 11. One obtains

$$\bar{A}_0^R(0) = 0.45 \text{ fm} \quad \text{and} \quad \bar{A}_0^I(0) = 0.40 \text{ fm}$$

for the incident proton,

$$\bar{A}_j^R(0) = 0.63 \text{ fm} \quad \text{and} \quad \bar{A}_j^I(0) = 0.50 \text{ fm}$$

for the outgoing protons originating from the 1p shell and

$$\bar{A}_j^R(0) = 0.66 \text{ fm} \quad \text{and} \quad \bar{A}_j^I(0) = 0.52 \text{ fm}$$

for the outgoing protons originating from the 1s shell.

Expression (2) for $P(\mathbf{q})$, using Eqs. (6) and the parameters above, has been calculated with the wave-functions $\psi_l^m(\mathbf{r})$ and the density $\rho(r)$ as determined by Eq. (7); these momentum distributions have been obtained for points at intervals of approximately 0.1 fm^{-1}. The radial functions, together with the radial distribution of the optical potential, are shown in Fig. 1a. Figures 1b and 1c show the corresponding distorted momentum distributions (full lines) and the undistorted ones ($D_0(\mathbf{r}) = D_1(\mathbf{r}) = D_2(\mathbf{r}) = 1$ in Eq. (3)), multiplied by the indicated factors (dashed lines).

In order to see the effect of using a matched distorting potential, a frequently performed type of non-matched calculation[14], namely using in Eq. (3) square well wave-functions ψ_l^m with a square well distorting po-potential, corresponding to a constant density in Eq. (5), has also been done. The root mean square radius of the distorting potential has been taken equal to the one of our matched potential, i.e., 2.64 fm for ^{16}O. From Eq. (5) follows that the volume integrals of both potentials are equal. The radial distribution of the potentials and the corresponding distorted momentum distributions are shown in Figs. 1 by the dot-dashed lines.

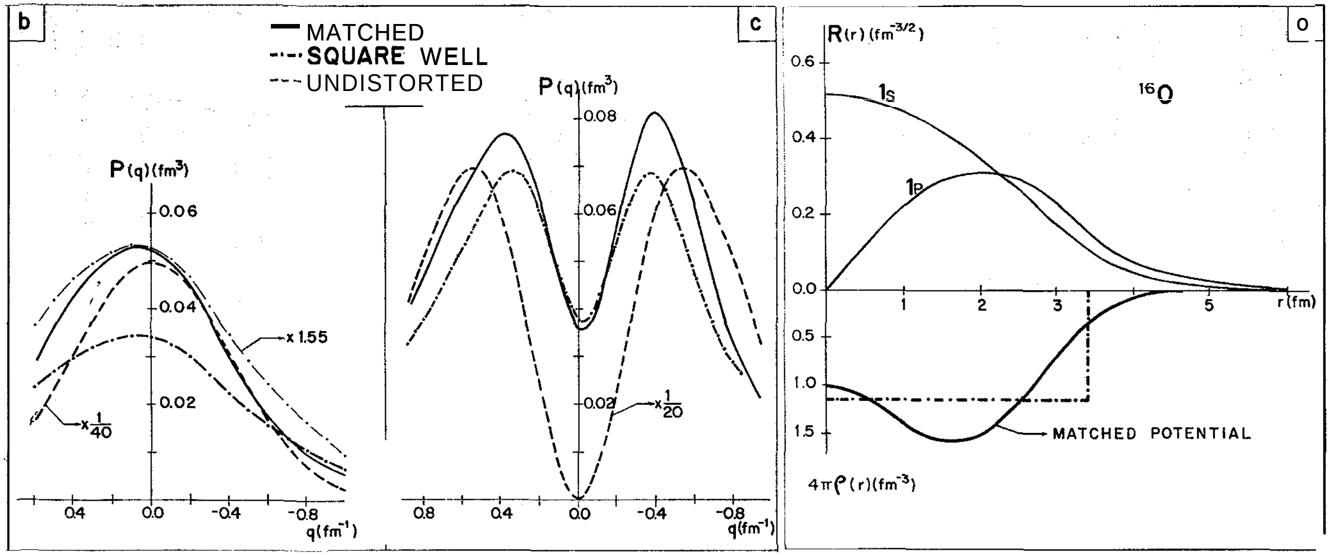

Figure 1. a) Wave functions for the square well potential and radial distributions of the matched and non-matched optical potentials; b) momentum distributions for 1*s* protons calculated with the indicated distorting potentials; c) same as b) for 1*p* protons.

To see better the effect in the shape of the 1s distorted momentum distribution, this distribution has also been plotted in Fig. 1b with a normalization factor of 1.55 (thin dot-dashed line).

3. Discussion

Comparing the results calculated using the matched potential with the ones obtained employing the uncorrelated square well potential (with the same root mean square radius) one observes that the effect on the s-state is more pronounced, both in shape and in magnitude, than the one on the p-state. This is understandable because for the s-state, which is located more centrally in the nucleus, multiple scattering effects are more important than for the p-state, located more at the surface, and so are all consequences of these effects. For this reason it is also to be expected that for nuclei heavier than the present very light ^{16}O nucleus, it will be even more essential to correlate the distorting potential with the wave-functions, i.e., to use the density obtained from the wave-functions in expression (5) for the distorting potential. Work on ^{40}Ca to confirm this statement is in progress.

Specifically, the main difference one observes in Fig. 1 is that the "matched" s-distribution is narrower and higher, which means a larger amount of low-momentum components; in particular, the decrease in the width is quite significant. Although to a smaller degree, similar effects are also present in the p-distribution.

We have repeated the complete calculation with a harmonic oscillator potential instead of a square well generating the wave-functions from which, as earlier, the distorting potential is derived. For comparison we used a Gaussian distorting potential with the same root mean square radius. Qualitatively, the results of these two calculations (matched and conventional) were similarly related as in the case just mentioned.

4. General Remarks on Quasi-Free Scattering

Quasi-free experiments have been a very useful tool for the study of certain overall properties of nuclei, as the separation energies and momentum distributions of the individual nuclear shells. The results of this paper show however that in the present status of the theoretical analysis of $(p,2p)$ experiments it is still too early to obtain detailed information on the parameters of models describing the nucleus; this would require better calculations of the distortion. As was convincingly shown in the experiments of Ref. 2 this distortion is, at least for light nuclei, to a remarkable extent independent of the energy sharing of the two outgoing protons. This fact justifies the hope that the results of a distorted wave analysis could be stated in a relatively simple manner.

One might hope that problems of detailed interpretation exist only for $(p,2p)$ experiments, where the distortion is serious, but not for $(e,e'p)$ measurements where the distortion can be taken into account by a constant reduction factor. As a matter of fact not only the distortion but also the description of the final state of the nucleus is in most studies still chosen so crudely that details of nuclear models are not really tested. As this important point seems not to be generally recognized, we make use of this opportunity to discuss it briefly and to give some examples.

The essential factor in the quasi-free correlation cross section is given, to a probably good approximation, by the overlap function of the initial and the final nucleus modified by the distortion resulting from the initial and final state interactions. Because there is one extra outgoing nucleon, this overlap function is, except for the distortion, the matrix element of the destruction operator corresponding to this nucleon (a real one and not a quasi-particle) between the initial and the final nucleus. As was pointed out in the original theoretical papers[15,8,16], it is only an approximation to take this overlap integral equal to the distorted single particle wavefunction of the knocked out particle and it is not surprising that in general only a semiquantitative agreement with experimental results[17,18,1-7] is achieved.

Several authors[19-21] have studied more detailed properties of the overlap integral, but in the literature the single particle picture is often taken very seriously even in drawing detailed conclusions from the experiments.

We believe however that for such conclusions to be warranted, the overlap integral has to be better understood. There are of course the modifications coming from the fact that the single particles should be taken to be quasi-particles, but at present we want to discuss another aspect which is more related to the typical reaction mechanism itself.

As long as one studies the angular correlations connected with a more or less sharp peak in the energy spectrum, one clearly selects those cases in which the final nucleus is in a more or less long lived state. This means that the nucleons in the final nucleus will have already adjusted to the absence of the knocked out nucleon, and therefore the overlap integral will deviate from the value given by the single particle wave-function. As one really compares different states of the final nucleus, resulting from a single state of the initial one, it is anyhow more natural to consider instead of the single particle state in the initial nucleus, the various states the hole can have in the final nucleus. This hole is moving in a complex single-hole potential[22,23] instead of in a single particle potential and already this simple improvement in the approximation gives the analysis new aspects of which we mention some.

a) The real part of the hole potential in the final nucleus is not necessarily identical to the single particle potential of the initial nucleus and therefore a direct cornparison of both seems to be a doubtful undertaking. Anyhow the finite lifetime of the hole state (as given by the energy width of the final nuclear state) will make the hole potential of an inner-shell state complex; this effect[22,23] alone may considerably influence the momentum wave-function of the hole which in this case is the overlap integral.

b) In the comparison of the separation energies of the inner shells, it is clear that one does not measure differences of binding energies of single particle states in the initial nucleus, but of single hole states in the final one[24]. Therefore it seems more natural to attempt to calculate directly the binding energies of the hole in the final nucleus than to compare the experimental results with single particle energies of the initial nucleus corrected with "rearrangement energies". From the observed widths in the energy spectra, one knows that this correction has an imaginary part which is strongly dependent on the shell consídered and the real part might have a similar dependence. This makes the comparison of calculated single particle energies with experiment very indirect and doubtful. In contradistinction, the complex energy of the hole state has an immediate experimental meaning.

c) Recently[25] the single particle picture has been taken so literally that even the Jastrow correlations which the knocked out particle had before it was ejected have been directly introduced into the single particle approximation of the overlap integral Predictably this procedure results in bumps in the momentum distributions at momenta corresponding to about the inverse of the hard core radius. We believe this effect not to be real and sketch the reason as seen from two points of view.

In the usual interpretation, taking the overlap integral equal to the Fourier transform of a single particle state, one should not forget that one wants the cross section in which the residual nucleus is in a certain, more or less stationary, state. The high momentum transfers corresponding to the Jastrow correlation of the knocked out particle will in general demand the taking up of this momentum by the correlated particle in the residual nucleus and will thus either lead to a prompt ejection of this nucleon or to an additional excitation of the residual nucleus. These events are in practice (and even in principle) hardly distinguishable from multiple scattering effects and will not contribute to the energy peak but to the smooth background in the energy spectrum. This argument restricting the effect of short range correlations was already given in the paper where the $(e,e'p)$ experiment was proposed.

In the less crude approximation of taking for the overlap integral the momentum distribution of the hole in the final nucleus, there seems to be no reason to expect that this hole in the *already rearranged* nucleus will remember that the missing particle had a hard core.

These arguments are based on the fact that one is studying one quasi--stationary state of the final nucleus in which the rearrangeinents have already taken place. Of course short range correlations may affect the cross section if a sum over the final nuclear states is performed.

Summarizing, we believe that the theoretical description of the distortion of the waves which represent the incoming and outgoing particles, as well as of the final nuclear states, have to be improved before reliable conclusions on details of nuclear models can be drawn from quasi-free experiments.

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