An Approximate Method for Studying Possible Collision Frequency and Temperature Changes in the D-Region during an SID

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Starting from the well known Wait and Spies exponential conductivity model of the lower ionosphere, an expression is derived for the change in the collision frequency in the lower ionosphere during an SID. The resulting change in the collision frequency is then combined with the expression for the collision frequency of mono-energetic electrons, given by Phelps and Pack, to derive an expression for the change in neutral temperature.

Partindo do bem conhecido modêlo exponencial de Wait e Spies para a condutividade da baixa ionosfera, foi obtida uma expressão para a variação da frequência de colisões na baixa ionosfera durante um SID. A combinação desta expressão com a expressão para frequência de colisôes de elétrons monoenergéticos, dada por Phelps e Pack, permite determinar variações na temperatura neutra daquelas regiôes.

1. Introduction

In recent years, considerable information has become available on the changes in the electron density in the D-region during a solar flare. All the information is based on either a) measurements using ground techniques, such as the partial reflection experiment[1] or b) using satellite and rocket measurements of the spectrum of the X-radiation emitted during the flare[2,3]. However, a survey of the available literature shows that there is little information on the collision frequency changes during an SID. Belrose[1] concludes from a survey of observational data, *that there is no evidence for collision frequencv changes during an SID, though short lived effects could have existed but been missed.* It is the purpose of the paper to derive a formula, to look for possible evidence in this direction in the height range 65-80 km using measurements on the amplitude and phase changes of VLF signals during the flare. It must be borne in mind however that as is generally the case with all indirect observations, the relation is at best an approximate one. The method of derivation of the formula, and its range of validity are discussed in Section 2. In Section 3, use is made of the results of Section 2 and the expression for the collision frequency of monoenergetic electrons given by Phelps and Pack[4] to derive an expression, for the change in neutral temperature.

2. The Collision Frequency Change during an SID

Wait and Spies[5] have shown that the conductivity parameter at VLF can to a fairly good degree of accuracy be approximated by an exponential formula of the form:

$$\omega_r = \omega_r(0) \exp\{\beta(h - h_0)\}, \tag{1}$$

where

$$\omega_r = \omega_0^2/\nu, \tag{2}$$

and

$$\omega_0^2 = 3.18 \times 10^9 \times N, \tag{3}$$

where ω_0 is the angular plasma frequency, N is the electron density (cm^{-3}) and ν is the collision frequency (sec^{-1}).

The quantity β in (1) is the gradient of the conductivity profile and has a value of 0.3 km^{-1} for the quiet daytime lower ionosphere. $\omega_r(0)$ is the conductivity parameter at the reference height h_0, and has a value of 2.5×10^5 at $h_0 = 70$ km during quiet conditions. The Wait and Spies exponential model has been successful in explaining the behaviour of the phase and amplitude of VLF signals observed during PCA events, and SID's. It has also been used to explain the anomalous behaviour of VLF signals on transequatorial paths, and has therefore found extensive use in the study of long distance VLF propagation phenomena. As such it would seem an ideal starting point for the present investigation. The validity of the model in the light of recent measurements of the electron density profiles in the D-region, is discussed at the end. Wait and Spies[5] have further shown that an adequate analytical form for the collision frequency is, (during quiet periods):

$$\nu = 5 \times 10^6 \, exp\{-0.15\,(h - 70)\}, \tag{4}$$

where h is expressed in kilometres.

The collision frequency of Equation (3) is the effective collision frequency. Equation (3) therefore takes into account, in an implicit fashion, the effect of the energy dependence of the collision frequency. Equations (1), (2) and (4) form the starting point for the present work. Using the conductivity profile of the lower ionosphere as given by (1), it is possible to obtain the relative changes in the conductivity parameter during an SID[6]. X-ray spectral information during the flare can be used to determine the change in electron density.

Let ω_{r_1}, N_1 , v, be the conductivity parameter, electron density and collision frequency before the flare and ω_{r_2}, N_2 , ν_2 the corresponding quantities at maximum flare. Then from (2) we have:

$$\Delta\omega_r = \omega_{r2} - \omega_{r_1} = \frac{KN_2}{\nu_2} - \frac{KN,}{\nu_1} \qquad (5)$$

or simplifying

$$\nu_2 = (KN_2\nu_1)/(\nu_1\Delta\omega_r + KN_1). \qquad (6)$$

In Equation (6), v,, N, and N_2 are known (N_2 from spectral data, and N_1 by combining Equations (2) and (3)). $\Delta\omega_r$ the relative change in the conductivity parameter can be evaluated from observed phase and amplitude changes during the flare. Thus all quantitites are known and v, can be evaluated. By making use of the phase and amplitude data on the same flare at two or three different frequencies, the collision frequency can be obtained as a function of height.

It is worthwile making an estimate of the errors involved in the determination of the electron density from X-ray spectral data, and the conductivity parameter from the phase and amplitude data in order to get an idea of the significance of v,, derived from (6).

Coming first to the problems of the determination of the electron density from X-ray spectral data, the question arises, as to how far the electron density calculated in this way agrees with the direct measurements. Somayjulu and Aikin (unpublished) have carried out simultaneous rocket measurements of changes in D-region ionization and solar X-rays during a solar flare event. The directly measured profile and that derived from X-ray spectral distribution show an agreement within about 20-25%, up to about 80 km.

The second point concerns the error in the determination of the conductivity. The main source of error in the determination of this quantity is the estimation of **Ah** based on **A&** readings, where **Ah** is the height change deduced from the phase advance in micro-seconds (A&).This latter quantity is read off the recordings, typically with an accuracy of ± 1 μsec leading to a maximum error of ± 5%. It is felt that a change in the collision frequency of 50% over its normal value as deduced from the above method, can be considered as significant.

3. The Temperature Change during an SID

By making use of laboratory data on the behaviour of slow electrons in Nitrogen and Oxygen[7] and the changes in collision frequency, it is in principle possible to obtain the changes in the neutral temperature resulting from the increased collision frequency.

As mentioned earlier, the collision frequency used in the theory so far, is the effective collision frequency ν_{eff}, which takes into account the energy dependence of the collision frequency. According to the generalized magnetoionic theory of Sen and Wyller[8] for a wave of angular frequency ω, the effective collision frequency ν_{eff} is related to the collision frequency of mono-energetic electrons ν, (of energy kT, k being Boltzmann's constant and T the absolute temperature) in a simple fashion for two limiting cases:

$$\begin{aligned} &\text{a)}\ \nu_{eff} \gg \omega, \quad \nu_{eff} \approx 2.5\nu_M ,\\ &\text{b)}\ \nu_{eff} \ll \omega, \quad \nu_{eff} \approx 1.5\nu_M . \end{aligned} \tag{7}$$

The collision frequency of mono-energetic electrons is after the work of Phelps and Pack[4] given by

$$\nu_M = (K_1 n_1 + K_2 n_2)u \quad [\sec^{-1}], \tag{8}$$

where

$$\begin{aligned} K_1 &= 1.12 \times 10^{-7},\\ K_2 &= 7.00 \times 10^{-8}, \end{aligned}$$

and n_1 and n_2 are the number densities of molecular Nitrogen and molecular Oxygen respectively and u is the electron energy. Combining Equations (4), (7) and (8), the electron energy u as a function of height can be evaluated. If now, the changes in the effective collision frequency is $d\nu$ we have. by differentiating (8),

$$d\nu_M = K_1 dn_1 u + K_2 dn_2 u + K_1 n_1 du + K_2 n_2 du. \tag{9}$$

Dividing (9) by (8), we get

$$\frac{d\nu_M}{\nu_M} = \frac{(K_1 dn_1 + K_2 dn_2)}{(K_1 n_1 + K_2 n_2)} + \frac{du}{u}. \tag{10}$$

The first term on the right hand side of (10) is negligibly small and can be set equal to zero. Hence (10) becomes.

$$\frac{d\nu_M}{\nu_M} = \frac{du}{u}. \tag{11}$$

Equation (11) can be used to evaluate du, since v,, dv, , and u are known. The electron energy u is related to the neutral temperature T (from kinetic theory of gases) by the relation

$$u = (3/2)\,R\,T \tag{12}$$

where R is the universal gas constant.

The change in neutral temperature can be evaluated using (12).

4. Discussion

The theory derived in the earlier sections is applicable to any SID event where simultaneous data on the X-ray spectral distribution and the phase and amplitude of the VLF signal are available. It is also applicable (obviously with greater accuracy) when direct measurements of the electron density during the flare are available. Before concluding, we wish to point out one more fact; this concerns the adoption of an exponential profile of electron density and collision frequency as being representative of quiet conditions. Wait and Spies[5] have made a comparative study of the exponential profile of collision frequency given by Equation (4). Between a height range of 65 and 85 km (the height range of interest in the present paper), the theoretical profile shows practically a one to one agreement with the experimental measurements of collision frequency by Belrose[9] and Kane[10]. It therefore seems quite reasonable to adopt an exponential profile of collision frequency. Coming now to the electron density, the same authors have compared profiles of ω_r (the conductivity parameter) given by Equation (1), with those given by Belrose and Burke[11] using the partial reflection experiment., by Barrington, Thrane and Bjelland[12] using the cross-modulation technique, and by Nicolet and Aikin[13], using photochemical theory. The general conclusion from this is that though the experimentally deduced profiles show some departure from the ideal form, it is not unreasonable to approximate the conductivity profile in the lower ionosphere with an exponential distribution and to regard more realistic models as a perturbation about the exponential form.

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